Strategic Investment Evaluation*

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Abstract

Motivated by stylized facts in the market for entrepreneurial fundraising, we study a setting in which two firms strategically time when to learn about the profitability of a project and when (if ever) to invest in it. Firms learn about the project privately, while investment decisions are public and provide a channel for social learning. Multiple equilibria exist, differing with respect to how much information firms acquire as well as how quickly they learn and invest. The equilibrium structure maximizing total firm profits varies with model parameters, implying testable predictions about the relationship between investor behavior and market conditions.

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1 Introduction

In many economic settings, decision makers may strategically delay action in order to observe and learn from the actions of others. This phenomenon of *strategic delay* arises when information about payoffs is dispersed, opportunities are non-rival, and decision-makers may

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freely time their actions. Our starting point is the observation that in many applications a decision maker's private information is the result of endogenous, costly information-acquisition activities. Strategic incentives then shape both how much information is *produced* through private effort, as well as how much is *aggregated* through public actions. Our goal is to understand the joint dynamics of information production and aggregation in settings featuring timing of both information acquisition and action.

A leading example is the market for entrepreneurial fundraising, in which venture capitalists evaluate startups for early-stage investment.[1] Many modern startups raise capital from multiple investors simultaneously, and are flexible as to the amount of capital raised, suggesting an environment of non-rival investment with significant risk and common payoffs.[2] Further, evaluating startups is a costly, time-consuming process involving multiple rounds of meetings with founders; review of documentation detailing the company's strategy, product, and financials; evaluation of the market and competing firms; and background checks of the startup's management team. In addition to producing their own information, there is also substantial anecdotal evidence that venture capitalists aggregate information through social learning, viewing startups as more appealing after they have received offers of funding from other investors. (In section 1.1 we provide evidence supporting these claims about the structure of the market for entrepreneurial fundraising.)

While the literature on investment timing suggests that firms may delay acting on information they have acquired, the literature on free-riding in teams indicates that firms may alternatively *free-ride*, or reduce their rate of evaluation of the project in order to exploit the effort of others. When both avenues of delay are available, their implications for firm behavior and profits are not clear ex ante. Our contribution is to build a tractable model to clarify the equilibrium interplay of incentives to delay information acquisition versus investment. Our model exhibits a parsimonious equilibrium set, yielding sharp characterizations of equilibrium behavior as well as novel insights into when symmetric versus asymmetric play maximizes total firm profits.

In our model two firms have the opportunity to invest in a nonrival risky project. Ex ante the project has negative expected returns. Firms may exert variable costly effort, a process we denote "prospecting", at each moment in time for the chance of receiving a binary signal

[1] Our model could also be applied to study product adoption by consumers or firm entry into a new market.

[2] The non-rival assumption may be inexact in some applications - for instance, funding rounds for very attractive startups may become oversubscribed and generate rivalry; and conversely firms in capital-intensive markets may exhibit increasing returns to scale from further investment. Still, we view the non-rival assumption as a good first approximation for initial analysis.

which is informative about the project's value. Each firm can acquire at most one signal, and signals are conditionally i.i.d. As a result, aggregating signals from multiple firms yields additional information about the profitability of investment beyond what any one firm could learn. Any information a firm acquires is private while investment is public, as in a standard strategic investment model. A key feature of our model is that effort yields information only stochastically. This creates a motive for acquiring information pre-emptively, because waiting until information becomes pivotal delays action. This benefit must be weighed against the cost of acquiring information that might be useful only far in the future, or never.

We show that there are exactly three perfect Bayesian equilibria of our model. In the unique symmetric equilibrium, each firm prospects as intensively as possible until a cutoff time, after which it abandons the project forever if it has not seen investment by the other firm. If at any time before the cutoff a firm receives a positive signal, it invests without delay. If it receives a negative signal it never invests. This equilibrium exhibits no free-riding or investment delay.[3]

There are also two asymmetric "leader-follower" equilibria. In these equilibria, one firm takes the role of a leader, prospecting as intensively as possible until acquiring a signal and then investing without delay if the signal is positive. Meanwhile the other firm, the follower, either shirks from acquiring a signal, delays investment after acquiring a signal, or both. The mix of free-riding and investment delay is controlled by the cost of prospecting. When prospecting costs are high, the follower eventually free-rides on the leader and does not work to acquire a signal, but does not delay investment on the equilibrium path. When costs are intermediate, the follower initially works to acquire a signal but delays investment if it obtains one, and eventually shirks if it does not. And when costs are sufficiently low, the follower waits for the leader to act after acquiring a signal but never shirks from acquiring one. In particular, when costs are low it is possible for the follower to acquire more information than it would have in the symmetric equilibrium.

We find that either equilibrium structure can generate higher total firm profits depending on model parameters. In general the symmetric equilibrium generates higher expected flow profits early in the evaluation phase, as both firms are prospecting and investing. But late in the evaluation phase the leader-follower equilibrium generates higher flow profits, as one firm continues evaluating the project past the time when both firms would have quit in the symmetric equilibrium. When firms are patient, the leader-follower equilibrium generates higher total firm profits, while when firms are impatient, the symmetric equilibrium

[3]By "free-riding" or "shirking" we mean that a firm does not prospect when it would be strictly optimal to do so in a one-player setting. "Investment delay" is defined similarly with respect to investment.

is superior.

Returning to the market for entrepreneurial fundraising, multi-market contact by venture capitalists across a portfolio of potential investments suggests a motive for coordinating on the total-profit-maximizing equilibrium when evaluating each project. Such coordination implies testable predictions for how investor behavior and evaluation periods change with market conditions, in particular as the promised return on investment needed to attract capital to a fund fluctuates. When the promised return is high, our model predicts evaluation times should be short but exhibit right-truncation, with startups rarely funded after long evaluation periods. Conversely, when the promised rate of return is low, the distribution of evaluation times should exhibit less right-truncation but shift outward, as some firms free-ride or delay investment. Additionally, evidence cited in Section 1.1 indicates that firms often adopt stratified roles in practice, with some firms leading evaluation and investment while other firms delay action until a leader has invested first. Our model suggests that such behavior may be most common in periods in which promised rates of return are low.

The remainder of the paper is organized as follows. Section 1.1 provides evidence on the structure of the entrepreneurial fundraising market, while Section 1.2 briefly surveys related literature. Section 2 illustrates several key features of our results via a simple three-period example. Section 3 describes the model. Section 4 characterizes the set of perfect Bayesian equilibria of the model. Section 5 studies the profitability of the different equilibria. Section 6 discusses how how our results would change under several generalizations of the model. Section 7 concludes.

1.1 Stylized facts about entrepreneurial fundraising

Here we provide evidence that the market for entrepreneurial fundraising exhibits the key economic forces present in our model: endogenous information acquisition; non-rival payoffs; and social learning through observation of investment.

Information acquisition is endogenous: Fried and Hisrich (1994) document that evaluation of a potential investment is an elaborate, multistage process requiring substantial active input by investment partners and analysts. Based on interviews with a small sample of venture capital firms, they estimate that due diligence takes on average 97 days, and 130 hours of cumulative effort by the lead investor, to complete for investments which are ultimately funded. A recent study by Gompers et al. (2019) surveying a much larger number of venture capitalists reaffirms these numbers, finding that venture capitalists take on average 83 days and 118 hours of effort to close a deal. Further, they report strong evidence that this evaluation process yields important information on the profitability of investment - only 1 in 20 startups evaluated are ultimately offered capital.

Payoffs are non-rival: Evidence from empirical analysis of venture capital contracts indicates that venture-backed startups typically accept capital from more than one firm during fundraising periods. Lerner (1994) analyzes a sample of biotech startups which obtained funding in the 1980s and found that on average between 2 and 4 venture capital firms participated in each of the first several rounds of fundraising for each startup. Kaplan and Strömberg (2003) study a larger sample of more recent startups, including biotech as well as IT and software firms, and find even larger rounds of between 3-6 firms in early rounds, with up to 10 firms in later rounds. The survey evidence of Gompers et al. (2019) reinforces these numbers, with 65% of reported investments by venture capitalists taking place as part of a syndicated (i.e. multi-investor) round. Further, a large fraction (75%) of investors cited capital constraints as an important reason for syndication, with 39% citing it as the single most important reason. This evidence suggests that many venture capitalists do not face significant congestion concerns when deciding whether to invest in a startup that has not yet secured capital from another fund.

Anecdotal evidence from practitioners reinforces this picture of non-rival investment. Paul Graham, a prominent entrepreneur and venture capitalist, advises entrepreneurs seeking funding that "It's a mistake to have fixed plans in an undertaking as unpredictable as fundraising... When you reach your initial target and you still have investor interest, you can just decide to raise more. Startups do that all the time. In fact, most startups that are very successful at fundraising end up raising more than they originally intended" (Graham (2013a)). This advice suggests that startups are often able to efficiently accept variable amounts of capital to meet unexpected demand.

Social learning is important: Lerner (1994), in his study of syndicated investing by venture capitalists, argues that social learning is so important that venture capitalists actively seek investing partners for the purpose of validating their investment decisions: "Another venture capitalist's willingness to invest in a potentially promising firm may be an important factor in the lead venture capitalist's decision to invest" (Lerner (1994, p. 16)). This assertion is reinforced by the survey evidence of Gompers et al. (2019), who report that 77% of venture capitalists surveyed cite "complementary expertise" of other investors as an important factor in deciding to join a syndicated round with multiple investors. (33% of respondents cited it as the single most important factor.)

Practitioners confirm the importance of social learning for investment decisions. Paul Graham writes that "The biggest component in most investors' opinion of you is the opinion of other investors... When one investor wants to invest in you, that makes other investors want to, which makes others want to, and so on." He elaborates that the cause of this phenomenon is social learning: "Judging startups is hard even for the best investors. The mediocre ones might as well be flipping coins. So when mediocre investors see that lots of other people want to invest in you, they assume there must be a reason" (Graham (2013b)).

This social learning motive is often discussed in conjunction with stratification of potential investor into active "leader" and passive "follower" roles. A handbook for fundraising, written by co-founders of a major venture capital fund, groups venture capitalists into "leader" and follower" categories, with leaders actively seeking to close a deal as a first investor while followers by contrast "han[g] around, waiting to see if there's any interest in your deal." Consistent with a social learning motive, they observe that followers are "not going to catalyze your investment. However, as your deal comes together with a lead, this VC is a great one to bring into the mix if you want to put a syndicate of several firms together" (Feld and Mendelson (2016, p. 25)).

1.2 Related literature

Our paper builds on two distinct literatures studying strategic investment timing and collective experimentation. Existing papers have typically studied only one of these forces in isolation. However, in reality decision-makers often have the flexibility to time both their investment and their information-gathering efforts. Our paper bridges this gap by modeling the two decisions jointly. We find that combining the two forces creates a novel tradeoff between free-riding and investment delay which has important implications for firm behavior and profits, particularly when selecting between symmetric and asymmetric equilibria.

Our paper builds most directly on the investment timing literature, in which multiple players decide when to make an irreversible investment in a risky project in the presence of social learning. The timing of investment is unrestricted and endogenous (in contrast to the herding literature), and information about the project's profitability is dispersed among the players, who can observe other players' investment decisions but not their information. This literature focuses on the strategic nature of the investment timing choice when agents learn from other agents' actions. One set of papers assume all private information is received at time zero. Papers in this tradition include Chamley and Gale (1994), Gul and Lundholm (1995), and Murto and Välimäki (2013). In all of these papers each player's information is exogenous. Aghamolla and Hashimoto (2020) endogenize the precision of the time-zero signal in the model studied by Chamley and Gale (1994) and Murto and Välimäki (2013), but do not allow agents to dynamically acquire any further information over the course of the game. A second set of papers assume, as we do, that information arrives gradually. Papers making this assumption include Chari and Kehoe (2004), Rosenberg et al. (2007), and Murto and Välimäki (2011). Unlike our model, these papers feature exogenous information arrival, and so abstract from the choice of when and whether to expend effort to learn about the project. Thus there is no tradeoff between free-riding and investment delay, a tension which plays a key role in our model.

Our paper is also related to models of collective experimentation, in which multiple agents engage in social learning by observing the outcome of repeated experimentation by other agents.[4] Bolton and Harris (1999), Keller, Rady, and Cripps (2005) and Keller and Rady (2010, 2015) assume that that each agent's experimentation choices are publicly observable. Bonatti and Hörner (2011, 2017) assume that actions are private, so that only payoffs are observed. Papers in this literature focus on the information spillovers of each agent's experimentation choices. Dong (2018) additionally studies the signaling effect of public experimentation when one player has private information about the state. All of the papers in this literature abstract from the timing of an irreversible commitment to one arm, a key ingredient of our model, as agents are able to flexibly choose their rate of experimentation at all times. The typical experimentation model yields a large number of equilibria, even with only two players and private actions. (One exception is Bonatti and Hörner (2017), who find a unique equilibrium with two players.) By contrast, our model yields exactly one symmetric and one asymmetric equilibrium, up to relabeling of players. As a result, we are able to make sharper predictions about behavior and welfare than do most experimentation models.

Our paper is also linked to several others which incorporate related assumptions on information acquisition or the strategic timing of investment. Guo and Roesler (2018) examine a strategic experimentation model with an option to quit permanently to secure an outside option. Akcigit and Liu (2016) analyze a patent race with two possible innovations, in which firms may privately and irreversibly switch lines of research after privately observing bad news about the viability of one line.[5] As in our model, firms in that paper learn via a signal acquisition technology in which "no news is no news", with each attempt to acquire information stochastically yielding a signal. Klein and Wagner (2019) build a model of strategic investment timing in which investment is reversible and players observe the outcomes of

[4]See Hörner and Skrzypacz (2017) for an excellent survey of this literature.

[5]They assume complete payoff rivalry, with only the first firm who achieves a particular innovation receiving a payoff. Thus the free-rider effect which plays an important role in our model is muted in theirs.

others' investments. Frick and Ishii (2020) study a model of investment timing in which investment boosts the arrival rate of public signals about the project's profitability. Ali (2018) studies the consequences of endogenous information acquisition in the context of a classic herding model, where players act in a pre-specified order. And Campbell et al. (2014) consider a setting of dynamic moral hazard in provision of a public good when each agent may unilaterally halt further effort by all agents and realize the current value of the good.

2 An illustrative example

Before presenting the full dynamic model, we illustrate several important features of our results in a simple three-period setting.

Consider two firms each deciding whether to irreversibly invest in a project, which is nonrival and can accept investment from both firms. The project's return is uncertain and could be either Good or Bad, with returns positive in the Good state and negative in the Bad state. In each of three periods $t = 1, 2, 3$, each firm can pay a cost $C > 0$ to attempt to learn about the project's profitability. Each attempt yields a binary signal $S \in \{H, L\}$ with probability $\Lambda \in (0, 1)$. In this example we will assume signals are perfectly revealing: A High signal reveals that the project is Good, while a Low signal reveals that the project is Bad. If a firm fails to acquire a signal, it learns nothing. Each firm can attempt to acquire a signal in every period if it has not yet succeeded. At the end of each period, after observing the results of their signal acquisition, each firm decides whether to invest or not. All signal acquisition is private, while past investment decisions are visible to the other firm. Firms discount future payoffs with discount factor $\delta \in (0, 1)$.

Let Π be the expected payoff to a firm from observing a signal for free and investing if that signal is High.[6] Because investment is profitable if the signal is High, $\Pi > 0$. Assume further that the project's expected returns without a signal are negative and that $\Lambda\Pi > C$, so that a signal is worth the cost of attempting to acquire. There exist two types of equilibria in this model: a symmetric equilibrium with frontloaded signal acquisition and an asymmetric equilibrium in which only one firm attempts to acquire a signal.[7]

In the symmetric equilibrium, both firms attempt to acquire a signal in period 1 and invest if they acquire a High signal. Assuming Λ is sufficiently large, in period 2 each firm becomes pessimistic about the project's quality absent a signal if they didn't see the other

[6] Using the notation of the main model, Π can be expressed as $\Pi = h(\pi_0)(R - 1)$, where $h(\pi_0)$ is the probability that an acquired signal is High, and $R - 1$ is the net payoff of a project that is known to be Good.

[7] We ignore the possibility of mixed-strategy equilibria, as they do not arise in the full model.

firm invest in period 1, because they infer that the other firm likely received a Low signal. Thus each firm stops attempting to acquire a signal after one period if they did not observe the other firm invest.

For such strategies to constitute an equilibrium, each firm must prefer to acquire a signal in period 1 rather than wait until period 2 to observe what the other firm has done. This incentive constraint may be written

$$\Lambda\Pi - C + \delta(1-\Lambda)\Lambda\Pi \geq \delta\Lambda\Pi.$$

The lhs is the payoff $\Lambda\Pi - C$ from attempting to acquire a signal and investing in period 1 if it is High, plus an additional payoff from investing in period 2 if they failed to acquire a signal in period 1 but the other firm invested. Meanwhile the rhs is the payoff to not acquiring a signal and investing in period 2 if the other firm invested in period 1. This equilibrium exists if C is not too high:

$$C \leq \overline{C} \equiv (1-\delta\Lambda)\Lambda\Pi.$$

In the asymmetric equilibrium, one firm shoulders the entire burden of signal acquisition, attempting to acquire a signal in every period, while the other firm does nothing, free-riding on the active firm's signal acquisition. These strategies constitute an equilibrium so long as the passive firm would rather wait to see what the active firm does rather than attempt to acquire a signal in period 1. This condition is

$$(\delta\Lambda + \delta^2(1-\Lambda)\Lambda)\Pi \geq \Lambda\Pi - C + (1-\Lambda)(\delta\Lambda + \delta^2(1-\Lambda)\Lambda)\Pi.$$

The lhs is the total expected payoff from not acquiring a signal but investing in periods $t = 2, 3$ if the active firm invested in period $t-1$. (Note that as period 3 is the final period, the passive firm does not benefit from observing the active firm's investment behavior at the end of that period.) Meanwhile the rhs is the total expected payoff from first attempting to acquire a signal in period 1, and if that fails becoming passive and waiting for the active firm to invest. This equilibrium exists if C is not too low:

$$C \geq \underline{C} \equiv (1-\delta\Lambda-\delta^2(1-\Lambda)\Lambda)\Lambda\Pi.$$

Observe that $\underline{C} < \overline{C}$. Thus when costs are moderate, both equilibria exist. A key question is which of the two equilibria achieves higher total firm profits. The two equilibria differ in both the total amount and the timing of signal acquisition. In the symmetric equilibrium, only two attempts are made to acquire a signal, but both these attempts happen immediately in period 1. Conversely in the asymmetric equilibrium, three total attempts are made to acquire a signal, but the second and third attempts occur in later periods. Which structure achieves higher total profits depends on the expected discounted amount of signal acquisition, which is a function of the discount factor δ.

One can verify numerically for specific sets of model parameters that the profit comparison may go either way as δ varies. For example, suppose $\Pi = 6$, $\Lambda = 0.5$, and $C = 2$. When $\delta = 0.55$, total profits in the symmetric equilibrium are

$$V^S = 2(\Lambda\Pi - C + \delta(1-\Lambda)\Lambda\Pi) = 3.65,$$

while in the asymmetric equilibrium total profits are

$$V^A = (\delta\Lambda + \delta^2(1-\Lambda)\Lambda)\Pi + (1 + (1-\Lambda)\delta + (1-\Lambda)^2\delta^2)(\Lambda\Pi - C) = 3.45 < V^S.$$

On the other hand, when $\delta = 0.65$ total profits in each equilibrium are $V^S = 3.95$ and $V^A = 4.01 > V^S$. In line with our earlier comparison of behavior in the two equilibria, total profits are higher under the asymmetric equilibrium when firms are patient, while profits are higher under the symmetric equilibrium when firms are impatient.[8]

One important simplification we have made in this example is to shut down the investment delay channel by assuming that signals are perfectly revealing, so that firms have no incentive to delay investment after acquiring a signal. Allowing signals to be partly informative introduces two forces which tend to favor asymmetric play, as in standard models of investment timing. First, the passive firm may be able to raise its payoff by buying a signal and then delaying action. Second, a symmetric equilibrium with full effort might not exist if delaying action is sufficiently profitable, further lowering the total discounted amount of signal acquisition.

Additionally, the finite-horizon, discrete-time setting of the example tends to mute incentives for free-riding that often lead to low levels of effort in frequent-action, infinite-horizon settings. In particular, the ability to flexibly delay effort over a long time horizon tends to undermine incentives for both firms to simultaneously exert full effort, an important feature of the symmetric equilibrium in our example.[9] Allowing for frequent actions over a long time horizon would therefore also tend to favor asymmetric play. It is important to analyze

[8]It can be checked that both equilibria exist at both discount factors under the given model parameters.
[9]See, e.g., Keller, Rady, and Cripps (2005) and Bonatti and Hörner (2011).

a model with these features to ensure that our insights generalize to richer environments.

Our analysis of the full model shows that the basic features of this example are still present when signals are partly informative and play takes place in continuous time over an infinite time horizon. In particular, analogs of the symmetric and asymmetric equilibrium studied above exist, and the symmetric equilibrium continues to exhibit full effort prior to abandonment. The profit comparison between the two equilibrium structures also displays the same comparative statics in the discount rate as in the example. Moreover, we show that in the full model, these are the only equilibria that can exist.

3 The model

Two firms have the opportunity to invest one unit of capital in a nonrival risky project of unknown quality. The project has underlying type θ and is either Good ($\theta = G$) or Bad ($\theta = B$). If $\theta = G$, each unit of capital invested in the project generates cashflows with a net present value of R, beginning at the time that unit of capital is invested; if $\theta = B$, the project generates no cashflows. We assume that $R > 1$, so that each unit of capital invested in the project generates positive returns in the Good state. Each firm is free to invest in the project at any time $t \in \mathbb{R}_+$. Firms are risk-neutral with common discount rate $r > 0$. Capital is indivisible, investment in the project is irreversible, and project outcomes are observed only by players who invest.[10]

Both firms begin with common prior belief π_0 that the project is Good. Each firm can additionally exert costly effort to search for an informative signal about the project's quality, an activity we will refer to as *prospecting*. A signal, when it arrives, is binary with $S \in \{H, L\}$, i.e. High and Low, and is distributed as $\Pr(S = H \mid \theta = G) = q^H$ and $\Pr(S = L \mid \theta = B) = q^L$ with $q^H, q^L \in (1/2, 1)$. For a given belief $\mu \in [0, 1]$ that $\theta = G$, let

$$h(\mu) \equiv q^H \mu + (1 - q^L)(1 - \mu)$$

be the total probability that an arriving signal is High, and similarly

$$l(\mu) \equiv (1 - q^H)\mu + q^L(1 - \mu) = 1 - h(\mu)$$

be the total probability that an arriving signal is Low. The values $h(\mu)$ and $l(\mu)$ are the transition probabilities that a firm's posterior belief jumps up or down upon receiving a

[10] A natural interpretation of the private observability of outcomes is that the project's cashflows are realized far in the future, as is common in many startups.

signal.

Each firm can obtain at most one signal, and firms observe conditionally i.i.d. signals. We will denote the posterior beliefs induced by one or more signals as follows: π_+ and π_{++} are the posteriors induced by one and two High signals, respectively; similarly π_- and π_{--} are the posteriors induced by one and two Low signals. Finally, π_{+-} is the posterior induced by one High and one Low signal. (Exchangeability implies that posterior beliefs are independent of the order of receipt of signals.) Given that High signals are more likely when the state is Good, and conversely for Low signals when the state is Bad, $\pi_{++} > \pi_+ > \pi_0, \pi_{+-} > \pi_- > \pi_{--}$. Note that in general $\pi_{+-} \neq \pi_0$, except in the special case when $q^H = q^L$. If $q^H > q^L$ then $\pi_{+-} < \pi_0$, and if $q^H < q^L$ then $\pi_{+-} > \pi_0$.

Assumption 1. $\pi_0 < 1/R < \pi_+$.

Under this assumption, investment in the project is ex ante unprofitable, but becomes profitable conditional on observation of a High signal.[11] Note that $1/R < \pi_+$ holds so long as q^L is sufficiently large, i.e. a High signal is sufficiently unlikely in the Bad state.

Assumption 2. $\pi_{+-} < 1/R$.

This assumption is satisfied so long as q^L is not too much smaller than q^H. Under this assumption, even after observing a High signal making investment profitable, observation of a Low signal would push beliefs back below the breakeven threshold. Without this assumption no equilibrium would exhibit investment delay, since the optimality of investment following receipt of a High signal would not depend on the information obtained by the other firm. (Note that $\pi_{+-} < 1/R$ does not inevitably imply investment delay, and indeed we will construct an equilibrium in which such behavior does not arise.)

Prospecting is a dynamic process unfolding in continuous time. At each instant dt, firm i's signal arrives with probability $\lambda\, dt$ when firm i exerts effort $C(\lambda)\, dt$. Following much of the literature on dynamic free-riding,[12] we assume a linear cost structure:

$$C(\lambda) = \begin{cases} c\lambda, & \lambda \in [0, \overline{\lambda}], \\ \infty, & \lambda \in (\overline{\lambda}, \infty) \end{cases}$$

[11] The case $\pi_+ < 1/R$ is uninteresting, as the unique equilibrium involves no prospecting and no investment by either firm. To see this, note that any firm investing first must have posterior beliefs weakly below π_+, meaning investment would be unprofitable. Hence no firm ever invests, and so never acquires a signal.

[12] See Bonatti and Hörner (2011) and Keller, Rady, and Cripps (2005) for classic examples of team experimentation models assuming linear experimentation costs.

for some constant marginal cost $c > 0$ and maximum prospecting rate $\overline{\lambda}$, both of which are symmetric across firms. Conditional on prospecting rates, signal arrival times are independent across firms and independent of the state of the project. Throughout our analysis we impose an upper bound on prospecting costs. Let $\overline{c} \equiv h(\pi_+)(\pi_{++}R - 1) - (\pi_+ R - 1)$.

Assumption 3. *$c \leq \overline{c}$.*

This assumption ensures that a second signal is at least potentially profitable to acquire, in the sense that if it could be attained instantaneously it would provide enough information to be worth the cost. As with Assumption 2, this assumption focuses our analysis on environments in which combining information from multiple signals is strategically relevant.[13] Note that Assumptions 1 and 2 ensure that $\overline{c} > 0$.

Firms cannot observe each other's signals or prospecting intensities, nor can they observe whether another firm has received a signal or obtained a good outcome from investment. There are also no communication channels between firms. However, all investment decisions are public, introducing a channel for social learning.

3.1 Strategies and payoffs

For each firm $i \in \{1, 2\}$, let s^i be the process tracking what signal, if any, firm i has observed at each moment in time. That is, $s^i_t \in \{\emptyset, H, L\}$ for each t, with $s^i_0 = \emptyset$ and s^i jumping at most once to either H or L at the time a signal is received. We will use $\nu^i \equiv \inf\{t \, : \, s^i_t \neq \emptyset\}$ to denote the time firm i receives a signal. Also let $\mathbb{F}^i$ be the filtration generated by s^i and a randomization device privately observed by i, with the latter allowing for mixed strategies.

A strategy σ^i consists of a family of $\mathbb{F}^i$-adapted prospecting processes $(\lambda^i(T)_t)_{t\geq 0}$ and investment processes $(\iota^i(T)_t)_{t\geq 0}$, one for each $T \in \mathbb{R}_+ \cup \{\emptyset\}$. The process $\lambda^i(T)$ describes firm i's prospecting behavior at each time supposing firm $-i$ has been observed to invest at time T (with $T = \emptyset$ indicating no investment so far), while $\iota^i(T)$ indicates whether firm i should invest at a given time supposing it hasn't yet done so. This notion of a strategy conditions on the other firm's investment activity through the dependence of the prospecting and investment rule on T; and it conditions on the firm's own signal, if any, through the $\mathbb{F}^i$-adaptedness of the prospecting and investment processes. The following definition summarizes our construction of a strategy.

Definition 1. *A strategy σ^i for firm $i \in \{1, 2\}$ is a tuple $\sigma^i = (\lambda^i(T), \iota^i(T))_{T\in\mathbb{R}_+\cup\{\emptyset\}}$, where for each investment time T of firm $-i$,*

[13] It can be shown that if this assumption is violated, investment delay never arises in equilibrium.

- $\lambda^i(T) = (\lambda^i(T)_t)_{t\geq 0}$ *is a* $[0, \overline{\lambda}]$*-valued* $\mathbb{F}^i$*-adapted prospecting process,*

- $\iota^i(T) = (\iota^i(T))_{t\geq 0}$ *is a* $\{0,1\}$*-valued* $\mathbb{F}^i$*-adapted investment process.*

A firm's strategy σ^i maps to a realized prospecting and investment history as follows. So long as the firm has not observed investment by firm $-i$, it prospects at rate $\lambda^i(\emptyset)_t$ at each time t until it obtains a signal, and invests at time $\tau^i(\emptyset) \equiv \inf\{t \, : \, \iota^i(\emptyset)_t = 1\}$. After i has observed $-i$ invest at some time T, the firm prospects at rate $\lambda^i(T)_t$ at each time $t > T$ until it obtains a signal, and it invests at time $\tau^i(T) \equiv \inf\{t \geq T \, : \, \iota^i(T)_t = 1\}$. This construction allows for the possibility that firm i, upon observing investment by firm $-i$, immediately follows and invests "afterward at the same time."[14] In particular, consider a strategy and state of the world in which firm 1 invests at time T. It will be important to allow for strategies for firm 2 under which $\tau^2(\emptyset) > T$, so that firm 2 would not invest at time T on its own, but under which $\tau^2(T) = T$, so that investment by firm 1 spurs firm 2 to act immediately.

Our definition of a strategy describes a firm's investment decision through the family of processes $\iota^i(T)$, which contain information beyond what is necessary to construct the investment time $\tau^i(T)$. This is because we will be interested in characterizing perfect Bayesian equilibria, which require a notion of optimality off the equilibrium path. Supposing that firm i has deviated and failed to invest at time $\tau^i(T)$, then at time $t > \tau^i(T)$ firm i's strategy induces the continuation investment time $\widetilde{\tau}^i(T) = \inf\{t' \geq t \, : \, \iota^i(T)_t = 1\}$. This allows for the important possibility that a firm who initially finds investment profitable may eventually become pessimistic and prefer not to invest immediately.

Fix a strategy profile $\sigma = (\sigma^1, \sigma^2)$. Firm i's expected payoff $U^i(\sigma)$ under σ is then

$$U^i(\sigma) = \mathbb{E}\left[(R\,\mathbf{1}\{\theta = G\} - 1)e^{-r\tau^i(\sigma)} - c\int_0^{\min\{\nu^i, \tau^i(\sigma)\}} e^{-rt}\lambda^i(\sigma)_t \, dt \right].$$

where $\lambda^i(\sigma)$ and $\tau^i(\sigma)$ are the on-path prospecting and investment rules under σ defined by

$$\lambda^i(\sigma)_t \equiv \begin{cases} \lambda^i(\emptyset)_t, & t < \tau^{-i}(\emptyset), \\ \lambda^i(\tau^{-i}(\emptyset))_t, & t \geq \tau^{-i}(\emptyset) \end{cases}$$

[14] In this respect, we follow the construction of strategy profiles used by Murto and Välimäki (2011), who model "exit waves" of firms who follow others out of the market with no delay. This model timing is necessary in continuous time to ensure existence of best replies. Otherwise a firm observing another investing/exiting might want to follow "as soon as possible", meaning any strategy of delaying a finite amount of time could be improved upon by delaying a bit less.

and

$$\tau^i(\sigma) \equiv \begin{cases} \tau^i(\emptyset), & \tau^i(\emptyset) \leq \tau^{-i}(\emptyset), \\ \tau^i(\tau^{-i}(\emptyset)), & \tau^i(\emptyset) > \tau^{-i}(\emptyset). \end{cases}$$

The first term in $U^i(\sigma)$ is the discounted payoff from investing in the project at time $\tau^i(\sigma)$, paying a cost 1 for the unit of invested capital and receiving net discounted cashflows of R iff the state is Good. The second term is the cumulative discounted cost of prospecting according to $\lambda^i(\sigma)$. The upper limit of integration reflects the fact that prospecting stops whenever either a signal arrives (at time ν^i) or the firm invests (at time $\tau^i(\sigma)$).

3.2 Beliefs

Given a strategy profile, let

$$\mu^i(t) \equiv \Pr(\theta = G \mid s^i_t = \emptyset, \tau^{-i}(\emptyset) \geq t)$$

be firm i's posterior beliefs at time t conditional on having seen no signal or investment so far, with

$$\mu^i_+(t) \equiv \Pr(\theta = G \mid s^i_t = H, \tau^{-i}(\emptyset) \geq t) = \frac{q^H \mu^i(t)}{h(\mu^i(t))}$$

and

$$\mu^i_-(t) \equiv \Pr(\theta = G \mid s^i_t = L, \tau^{-i}(\emptyset) \geq t) = \frac{(1 - q^L)\mu^i(t)}{l(\mu^i(t))}$$

similarly representing firm i's beliefs conditional on having observed a High and Low signal, respectively.[15]

In contrast to Poisson bandit models, under our prospecting technology each firm's posterior beliefs about the state are independent of their own history of prospecting. Lack of signal acquisition in our model does not signal anything, positive or negative, about the true project state; no news truly is no news until a signal arrives.[16] However, firms do update their beliefs due to social learning, as they can observe whether the other firm has invested. Consequently, if firm i is prospecting and investing, then firm $-i$'s beliefs $\mu^{-i}(t)$ deteriorate over time due to the negative inference from continued lack of investment by the other firm. The longer firm i stays out, the more convinced firm $-i$ becomes that i is dormant due to

[15] Lemma O.1 in the online appendix shows that these beliefs are always uniquely pinned down by Bayes' rule in any PBE strategy profile. Note that if $\tau^{-i}(\emptyset) = t$, then firm i is not able to observe this fact until after making his own initial investment decision, so its beliefs at time t cannot condition on this fact. Hence the appropriate conditioning for "no investment by firm $-i$ up to time t" is the event $\tau^{-i}(\emptyset) \geq t$.

[16] A similar signal acquisition technology is employed in Akcigit and Liu (2016).

receipt of a Low signal, rather than a long string of bad luck leading to no signal.[17]

Of course, the rate at which beliefs deteriorate is an endogenous property of a particular equilibrium, and in particular will depend on whether each firm expects the other to prospect and invest, versus shirk or wait to invest. This linkage of beliefs about the state and the (unobserved) strategy of one's opponent will play a crucial role in the construction of equilibria in this setting.

3.3 Single-player benchmark

Consider a single firm prospecting and investing on its own, shutting down the social learning channel of our model. We will refer to this benchmark setting as the *autarky case*. The firm's initial beliefs that the project is good will be taken to be $\mu < 1/R$.

So long as the firm has acquired no signal, it learns nothing about the project and its beliefs remain fixed at μ. An optimal prospecting strategy is therefore stationary. (This behavior contrasts with the cutoff strategies which are optimal when learning from Poisson bandits, and is driven by the difference in learning dynamics as discussed in Section 3.2.) Once the firm has acquired a signal, no further information is available. It then faces a simple choice of whether to invest or abandon the project once and for all, by comparing its posterior beliefs to the investment threshold $1/R$.

The optimal prospecting strategy depends on whether the firm's initial beliefs μ lie above a critical threshold, which we will denote π_A and refer to as the *autarky threshold*. It is formally characterized as the unique solution to the equation

$$h(\mu)(\mu_+ R - 1) = c,$$

which equalizes the marginal flow gains and costs from a unit of prospecting. Some algebra shows that π_A has the explicit representation

$$\pi_A = \frac{c + (1 - q^L)}{q^H(R-1) + (1 - q^L)}.$$

If $\mu < \pi_A$, the firm abandons the project immediately and does not prospect. On the other hand, if $\mu > \pi_A$, then the firm prospects at the maximum rate $\overline{\lambda}$ until a signal is acquired.

[17] Our assumption that each firm can acquire at most one signal simplifies this inference problem by ensuring that the other firm's private information can take exactly two forms — uninformed or informed with a Low signal. If firms could acquire multiple signals, each possible set of acquired signals would have to be taken into account when calculating posterior beliefs.

If firms' initial beliefs π_0 lie below π_A, then even in a model with social learning no prospecting or investing will ever take place.[18] Going forward we will assume that this is not the case. In particular, since π_A is increasing in the cost parameter c, we will assume that prospecting costs are low enough that firms would want to prospect in the single-player benchmark.

Assumption 4. *c is sufficiently small that $\pi_0 > \pi_A$.*

Note that as c approaches 0, π_A approaches the prior belief $\underline{\mu}$ for which $\underline{\mu}_+ R - 1 = 0$, where $\underline{\mu}_+ = q^H \mu / h(\mu)$. Since by assumption $\pi_+ R - 1 > 0$, it must be that $\pi_0 > \pi_A$ for sufficiently small c.

3.4 Free-riding and investment delay

Throughout this paper we will be interested in characterizing when firms delay information acquisition and investment. We now formally define these notions in the context of our model:

Definition 2. *Given a strategy profile σ, firm i :*

- Free-rides *or* shirks *at time t if $\mu^i(t) > \pi_A$ and $\lambda^i(\emptyset)_t < \overline{\lambda}$.*
- Abandons the project *at time t if $\mu^i(t) \leq \pi_A$ and $\lambda^i(\emptyset)_t = 0$.*
- Delays investment (on the equilibrium path) *if*

$$\Pr(\tau^i(\sigma) > \nu^i \text{ and } s^i_{\nu^i} = H \text{ and } \mu^i_+(\nu^i) > 1/R) > 0.$$

We consider a firm to be free-riding (or, equivalently, shirking) if, prior to observing the other firm invest, it exerts effort strictly below what would be optimal in a one-player setting at current beliefs. Any such effort reduction must be because the firm expects to learn from the other player's future actions, a motive typically labeled shirking in the literature on free-riding in teams. (See, for instance, Bonatti and Hörner (2011).) By contrast, we consider the firm to have abandoned the project if, prior to observing the other firm invest, it stops prospecting at beliefs sufficiently low that this would be the optimal action in a one-player

[18] In particular, there can be no encouragement effect inducing prospecting below the single-player cutoff in this model. This is because in equilibrium some firm must invest first after obtaining a signal, and that firm does not benefit from inducing the other firm to act after it.

setting. Note that abandonment is a conditional decision—were the firm to see the other firm invest, its beliefs would jump upward and it might optimally resume prospecting.

Similarly, we consider a firm to be delaying investment on the equilibrium path if, prior to observing the other firm invest, it sometimes obtains a High signal which 1) boosts its beliefs above the threshold for profitable investment, and 2) the firm does not immediately act on. This definition of investment delay is consistent with the convention used in the investment timing literature. (See, e.g., Chamley and Gale (1994).) We will typically omit the qualifier "on the equilibrium path" when we expect no confusion. Our concept of delay excludes instances which occur off the equilibrium path, focusing our analysis on observable firm behavior.

Note that free-riding, abandonment, and investment delay are positive, not normative labels. In Section 5 we examine how total firm profits are impacted by the free-riding and investment delay arising in different equilibria.

4 Equilibrium analysis

In this section we characterize the set of perfect Bayesian equilibria of the model. We find that our model has exactly three equilibria. One equilibrium is symmetric and exhibits no free-riding or investment delay, but does lead to eventual abandonment of the project by both firms. The remaining "leader-follower" equilibria feature distinct roles for the two firms, with one firm who takes the lead in prospecting and investing while the other firm plays a passive follower role. In general these equilibria feature either free-riding, investment delay, or both by the follower, with the mix shifting from free-riding toward investment delay as prospecting costs fall.

The section is structured as follows. In Section 4.1 we describe each firm's optimal continuation strategy after observing investment by the other firm. In Section 4.2, we describe a class of threshold strategies for prospecting and investing prior to observing the other firm invest which arise in every equilibrium. In Sections 4.3 and 4.4 we characterize the symmetric and leader-follower equilibria and provide intuition for their properties. In Section 4.5 we prove that no other equilibria exist.

4.1 Behavior after observing investment

One important aspect of equilibrium behavior is the optimal continuation strategy of a firm after observing the other firm invest. It can be shown that in any equilibrium, the first firm to invest is always in possession of a High signal.[19] The remaining firm therefore finds itself in a stationary single-player environment with fixed beliefs, analogous to the autarky benchmark studied in Section 3.3. If the firm has already acquired a signal, these beliefs are either $\pi_{++} > 1/R$ or $\pi_{+-} < 1/R$, and no further information can be acquired. The firm therefore either invests immediately, if it is in possession of a High signal, or abandons the project if it is in possession of a Low signal.

On the other hand, if the firm has not yet acquired a signal, its beliefs are $\pi_{+} > 1/R$ and it has the opportunity to acquire a signal before investing. It therefore either invests immediately or prospects at rate $\overline{\lambda}$ until it obtains another signal, depending on the firm's patience.[20] The following lemma states this result formally. (The proof is straightforward, and so is omitted for brevity.)

Lemma 1. *There exists a threshold discount rate $r^* > 0$ such that under any strategy profile σ supportable in a perfect Bayesian equilibrium:*

- *If $r \leq r^*$, each firm i's strategy satisfies $\lambda^i(\sigma)_t = \overline{\lambda}$ and $\iota^i(\sigma)_t = \mathbf{1}\{s_t^i = H\}$ subsequent to observing investment with probability 1 under σ.*
- *If $r > r^*$, each firm i's strategy satisfies $\iota^i(\sigma)_t = \mathbf{1}\{s_t^i \neq L\}$ subsequent to observing investment with probability 1 under σ.*

(Note that in the high-discount-rate case, the firm's choice of prospecting strategy after observing investment is not payoff-relevant and need not be specified.) The size of r relative to r^* impacts continuation payoffs and the speed of follow-on investment, but otherwise does not substantially impact the structure of the equilibrium set.

4.2 Threshold strategies

With Lemma 1 in hand, the remaining aspects of firm behavior to be described are prospecting and investment behavior prior to observing investment by another firm. In principle, there are diverse possibilities for such behavior. For instance, as in the literature on strategic experimentation, firms might prospect at declining rates over time, shirk for a time before

[19] See the online appendix for a formal derivation of this result.

[20] Formally, such behavior must arise following *on-path* investment by the other firm. If investment at a particular time by the other firm was off-path, our equilibrium notion allows the firm to freely form beliefs about what signal the other firm possessed, potentially supporting other continuation strategies. However, the strategy chosen by a firm in such off-path information sets affects neither the incentives of the other firm (who does not care what their rival does following their own investment) or equilibrium payoffs. As a result, we ignore this multiplicity.

beginning to prospect, or even alternate between periods of high effort and shirking so long as no investment has been observed.[21] And as in the literature on strategic investment, firms might randomize over the time at which they invest following receipt of good news about the project.

It turns out that in our model, none of these possibilities can arise in equilibrium. Rather, both prospecting and investment behavior exhibit a simple bang-bang structure, with prospecting switching at most once from rate $\overline{\lambda}$ to 0, and investment switching from immediate investment to maximal delay.

Definition 3. *A strategy σ^i for firm $i \in \{1,2\}$ is a* threshold strategy *if there exist thresholds $\overline{T}_i, T_i^* \in \mathbb{R}_+ \cup \{\infty\}$ such that $\lambda^i(\emptyset)_t = \overline{\lambda}\mathbf{1}\{t < \overline{T}_i\}$ and $\iota^i(\emptyset)_t = \mathbf{1}\{s_t^i = H \text{ and } t < T_i^*\}$.*

If firm i employs a threshold strategy, it prospects at the maximum possible rate $\overline{\lambda}$ until the threshold time $\overline{T}_i$. At such times we will say that firm i is *working*. Meanwhile after $\overline{T}_i$ the firm quits prospecting. Whether this behavior constitutes shirking or abandonment depends on the firm's beliefs at a given time. Let T_i^A be the time at which firm i's beliefs drop to π_A. If $\overline{T}_i < T_i^A$, then firm i shirks on the interval $[\overline{T}_i, T_i^A]$. By contrast, after time $\max\{T_i^A, \overline{T}_i\}$ firm i abandons the project. Meanwhile prior to time T_i^*, the firm invests immediately whenever it is in possession of a High signal (on or off the equilibrium path), while after T_i^* it never invests prior to the other firm investing. We will refer to the period prior to T_i^* as the *investment* phase, and the period subsequent to T_i^* as the *waiting* phase.

As we will see, in every equilibrium both firms follow threshold strategies. This fact, along with Lemma 1, reduces the description of an equilibrium to the characterization of the threshold times $\overline{T}_i$ and T_i^* for each firm.

4.3 The symmetric equilibrium

In this subsection we characterize the unique symmetric equilibrium of the model. This equilibrium exhibits no free-riding or investment delay, but does involve eventual abandonment of the project by both firms.

To state the equilibrium, we define a time threshold at which a firm's posterior beliefs hit π_A assuming the other firm does not delay signal acquisition or investment. Suppose that some firm i prospects at rate $\overline{\lambda}$ forever and invests immediately whenever it obtains a High

[21] Each of these behaviors arises, for instance, in equilibria of Keller, Rady, and Cripps (2005). Note in particular that an assumption of linear experimentation costs, as we maintain, does not in general rule out interior equilibrium effort. See also Bolton and Harris (1999), Bonatti and Hörner (2011), and Keller and Rady (2010) for further examples of models with linear experimentation costs whose equilibria exhibit interior experimentation rates.

signal. Let $\mu^{\overline{\lambda}}$ denote the associated path of firm $-i$'s beliefs conditional on observing no investment by firm i.[22] These beliefs decline over time, asymptotically approaching π_-, and cross the autarky threshold π_A at some finite time which we will denote $T^A \equiv (\mu^{\overline{\lambda}})^{-1}(\pi_A)$.

Proposition 1 (The symmetric equilibrium)**.** *There exists a symmetric perfect Bayesian equilibrium in threshold strategies characterized by* $\overline{T}_1 = \overline{T}_2 = T^A$ *and* $T_1^* = T_2^* = \infty$.

This equilibrium unfolds as follows. Absent observing investment by the other firm, each firm works, i.e. prospects at rate $\overline{\lambda}$, until time T^A. Afterward each firm abandons prospecting forever. If at any time a firm observes investment, it follows the optimal continuation strategy characterized in Lemma 1. If at any time a firm is in possession of a High signal (on or off the equilibrium path), it invests immediately. In other words, each firm is in the investment phase forever. Finally, no firm invests while in possession of no signal or a Low signal. The structure of prospecting and investing prior to observing investment by the other firm is represented diagrammatically in Figure 1. Note that the equilibrium is symmetric — given the structure of threshold strategies and the optimal continuation play characterized by Lemma 1, equal prospecting and investment thresholds imply symmetric strategies.

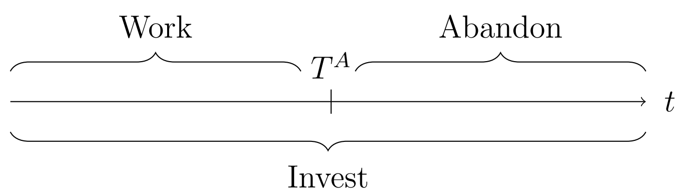

Figure 1: A timeline of prospecting and investment in the symmetric equilibrium

One key feature of this equilibrium is that it exhibits *neither free-riding nor investment delay*. In particular, at no point in time does a firm shirk from acquiring a signal while its beliefs are above π_A, nor does any firm in possession of a High signal ever delay investment.

Another important feature is that *the project is eventually abandoned*. If by time T^A no firm has invested, both firms cease efforts to acquire a signal forever afterward.[23] Note that after time T^A, each firm is indifferent between prospecting or not, as their beliefs remain fixed at π_A forever afterward. However, abandonment is the unique continuation strategy that can be sustained as part of an equilibrium. For it is precisely the lack of information arriving after beliefs reach π_A which makes it optimal for firms to prospect at all times prior

[22]We explicitly calculate these beliefs in the online appendix.

[23]This effect resembles the investment collapse phenomenon described in Chamley and Gale (1994). In that paper a collapse is precipitated by randomization over investment at early stages of the game. By contrast, in our setting abandonment is induced by stochastic information acquisition.

to T^A. If some firm were to continue prospecting, they would drive the other firm's beliefs below π_A in finite time, and that firm would then no longer optimally prospect until time T^A. The dynamics of belief updating in this equilibrium prior to obtaining a signal are illustrated in Figure 2.

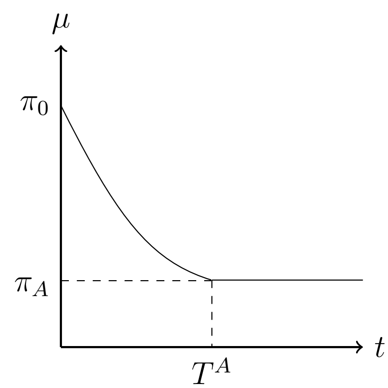

Figure 2: Evolution of beliefs in the symmetric equilibrium

In the remainder of this subsection, we provide some intuition for the optimality of each firm's strategy in this equilibrium. First consider each firm's investment strategy. Because both firms quit prospecting at time T^A, each firm's beliefs are always at least π_A for all times, even absent a signal or any observed investment. This means that after obtaining a High signal, each firm's beliefs lie above $1/R$ forever. So by waiting to invest, no firm ever obtains enough negative information to change their optimal investment decision, meaning any delay in investing is suboptimal.

Now consider the equilibrium prospecting rule. The optimality of the rule is most straightforward when firms are patient, i.e. $r \leq r^*$ and firms acquire their own signal after observing the other firm invest. In this regime each firm never learns anything from the other firm's actions which is pivotal to their decision to prospect or invest. In particular, consider continuations in which the other firm has or has not invested. In the first case, it continues to be optimal to acquire another signal. And in the second case, the firm's beliefs deteriorate over time, but critically not past the threshold level π_A given the cutoff time T^A. Thus no information ever arrives which makes the firm so pessimistic about the project that signal acquisition isn't optimal. The result is that in all continuations, whether the other firm has invested or not, each firm's optimal prospecting decision remains the same as in the autarky case. It is then certainly optimal to prospect at rate $\overline{\lambda}$ prior to time T^A. Subsequent to T^A the firm is made indifferent between prospecting or not, so it is (weakly) optimal to cease prospecting at that point.[24]

When $r > r^*$, social learning *does* have value for each firm even prior to time T^A, as observing investment allows them to save on further prospecting costs by investing immediately afterward.[25] There is then a potential incentive for firms to shirk in order to free-ride on the prospecting of the other firm. It turns out that when Assumption 3 holds, this incentive is never strong enough to induce shirking.

4.4 The leader-follower equilibrium

In this subsection we characterize a pair of asymmetric equilibria in which one firm takes the lead to prospect and invest. Unlike the symmetric equilibrium, in these equilibria the project is never abandoned by both firms: with probability 1 at least one firm eventually acquires a signal about the project. However, they feature free-riding, investment delay, or both by the non-lead firm. Throughout this section, we describe the equilibrium in which firm 1 is the leader and firm 2 is the follower. By symmetry, another equilibrium exists with the roles of the firms reversed.

Proposition 2 (The leader-follower equilibrium)**.** *There exist unique belief thresholds* $\overline{\mu}, \mu^* \in (\pi_-, \pi_0]$ *such that the threshold strategies* $\overline{T}_1 = T_1^* = \infty$ *and* $\overline{T}_2 = (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$, $T_2^* = (\mu^{\overline{\lambda}})^{-1}(\mu^*)$ *constitute a perfect Bayesian equilibrium. Further,* $\max\{\overline{\mu}, \mu^*\} > \pi_A$.

(Recall that $\mu^{\overline{\lambda}}$ is the posterior belief process of a firm who believes its opponent prospects at rate $\overline{\lambda}$ and invests immediately.)

This equilibrium unfolds as follows. Firm 1 takes the role of the *leader*, and works (prospects at rate $\overline{\lambda}$) until it obtains a signal so long as it has not seen the follower invest. If at any time the leader is in possession of a High signal (on or off the equilibrium path), it invests immediately. It is therefore in the investment phase forever. Meanwhile firm 2 takes the role of the *follower*. Absent observing investment, the follower works only up until the threshold time $\overline{T}_2 < \infty$. After this time it stops prospecting until it observes investment. If at any time t the follower is in possession of a High signal (on or off the equilibrium path), it invests immediately if and only if $t < T_2^*$, and otherwise it waits until it observes investment by the leader. Because $\max\{\overline{\mu}, \mu^*\} > \pi_A$, the follower becomes passive, i.e. ceases investing before the other firm on the equilibrium path, earlier than it would have in the symmetric

[24] An immediate consequence of this logic is that when $r \leq r^*$, each firm's equilibrium payoff in the symmetric equilibrium is exactly their autarky payoff.

[25] As a consequence, when $r > r^*$ each firm's symmetric equilibrium payoff strictly exceeds their autarky payoff.

equilibrium. If either firm observes investment when not in possession of a signal, it follows the optimal continuation strategy characterized in Lemma 1.

The structure of prospecting and investing prior to observing investment by the other firm is represented diagrammatically in Figure 3. The evolution of each player's belief prior to obtaining a signal is illustrated in Figure 4. Both figures illustrate an equilibrium in which the follower is initially active, i.e. $\min\{T_2^*, \overline{T}_2\} > 0$. Further, Figure 3 illustrates an equilibrium in which the follower stops investing before it stops prospecting, i.e. $\overline{T}_2 > T_2^*$, and stops prospecting before time T^A. Recall that whenever a firm is not prospecting, we have defined such behavior as *shirking* if that firm would optimally prospect in a single-player setting at the same beliefs, and as *abandonment* otherwise. In the leader-follower equilibrium, shirking gives way to abandonment at time T^A, when the follower's beliefs drop below π_A.

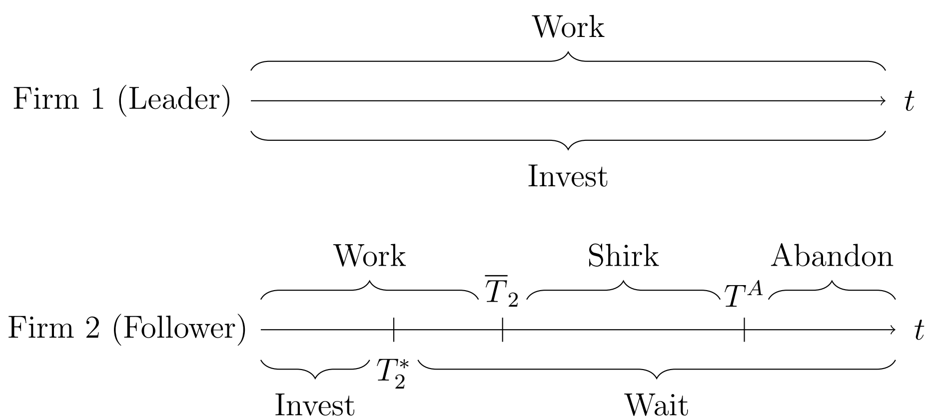

Figure 3: A timeline of prospecting and investing in the leader-follower equilibrium

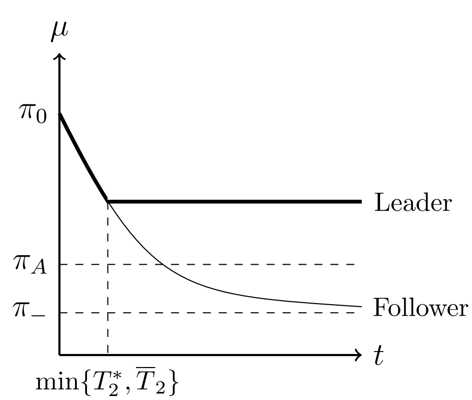

Figure 4: Evolution of beliefs in the leader-follower equilibrium

Unlike the symmetric equilibrium of Proposition 1, the leader-follower equilibrium exhibits shirking, investment delay, or a combination of the two. The following lemma establishes when each arises in equilibrium as a function of the cost of prospecting c.

Lemma 2 (Comparison of thresholds)**.** *There exist thresholds c^*, c_*, c'_* satisfying $\overline{c} \geq c^* > c_* \geq c'_* > 0$ such that in the leader-follower equilibrium:*

- *If $c > c^*$, then $\overline{T}_2 \leq T_2^*$, while if $c < c^*$, then $T_2^* < \overline{T}_2$.*
- *If $c > c_*$, then $\overline{T}_2 < T^A$, while if $c < c'_*$, then $\overline{T}_2 > T^A$.*

The first part of the lemma establishes existence of a threshold c^* for prospecting costs above which the follower quits prospecting before it quits investing, and below which it continues prospecting even after the time at which it would stop investing following receipt of a High signal. Thus investment delay emerges on the equilibrium path only when prospecting costs are sufficiently low. The second part of the lemma establishes existence of a second threshold c_* for prospecting costs above which the follower eventually shirks, that is, fails to exert effort when it would be optimal at current beliefs in a one-agent setting. Below the cost threshold $c'_* \leq c_*$, the follower not only exerts effort until its beliefs fall to π_A, but in fact exerts effort at beliefs low enough that effort would be strictly suboptimal in the one-player problem. In this case the option value effect from delaying investment is so large that it actually makes signal acquisition more profitable for low beliefs than in the absence of the ability to delay.

The fact that $c'_* \leq c_* < c^*$ implies that equilibrium behavior moves through three distinct regimes as prospecting costs drop. When costs are high, above c^*, the follower free-rides but never delays investment.[26] When costs are intermediate, between c_* and c^*, the follower delays investment early on in the project and eventually shirks later in the project. Finally, when costs are low, below c'_*, the follower delays investment but never shirks.[27] Thus as prospecting costs drop, the mix of free-riding and investment delay shifts away from free-riding and toward delay.

In the remainder of this subsection, we provide some intuition for the optimality of each firm's strategy in this equilibrium. Consider first the leader's strategy. Subsequent to time

[26]The proof of Lemma 2 establishes that whenever r is sufficiently small, $c^* < \overline{c}$ and this regime is non-degenerate.

[27]Whenever $c_* = c'_*$, follower behavior moves through exactly three regimes as c varies. In the proof of Lemma 2, we establish a number of sufficient conditions for this equality to hold: for instance, it holds whenever r is sufficiently small or sufficiently large, or if $h(\pi_+)(\pi_{++}R - 1) \geq 2(\pi_+ R - 1)$. Further, $c_* = c'_*$ in practice in every numerical example we have checked across a wide range of parameter values. We conjecture that $c_* = c'_*$ in general.

$\widehat{T}_2 \equiv \min\{\overline{T}_2, T_2^*\}$, the follower is passive and the leader is effectively in autarky with beliefs $\mu^{\overline{\lambda}}(\widehat{T}_2)$. Since $\widehat{T}_2 < T^A$, the leader's unique optimal continuation strategy at time $\widehat{T}_2$ is to actively prospect and invest as in the single-player benchmark. Prior to time $\widehat{T}_2$, the follower is active and there is some incentive for the leader to delay in order to observe what the follower does. However, this incentive is always weaker than the incentive for the follower to delay in order to observe the leader, since the follower expects more future action by the leader than does the leader from the follower. Thus since the follower is optimally active at such times (as we argue below), so is the leader.

The follower's optimal strategy can be characterized in two stages by backward induction. First, consider information sets in which the follower has obtained a High signal. At this point the follower faces an optimal stopping problem of when to invest based on its information in the absence of action by the leader. As time progresses, the follower's posterior beliefs deteriorate and the value of immediate investing declines relative to the option value of waiting for more information. At the belief threshold μ^*, which is uniquely determined, the value of immediate investing is just equal to the value of waiting an instant. Thus for beliefs above this level, i.e. times before T_2^*, immediate investing is optimal. On the other hand for times after T_2^*, waiting an instant further is always preferable to investing immediately, and the follower refrains from investing at any point prior to the leader acting.

The follower's optimal prospecting strategy can then be similarly determined, taking into account the optimal use of a High signal should one be acquired. The tradeoff is fundamentally similar to the investment delay problem, with the value of immediately acquiring a signal declining over time relative to the option value of waiting an instant for more information. At a uniquely determined belief threshold $\overline{\mu}$, these two values coincide, so that for times prior to $\overline{T}_2$ prospecting is strictly optimal, while for times subsequent to $\overline{T}_2$ shirking is strictly optimal. Further, at least one of μ^* and $\overline{\mu}$ must lie strictly above π_A, as otherwise the follower would be prospecting when the value of obtaining a signal is approximately zero, which cannot be optimal given the strictly positive option value of waiting for information from the leader's actions.

4.5 Characterization of the equilibrium set

So far we have demonstrated the existence of three equilibria: a symmetric equilibrium and two leader-follower equilibria (which are identical up to permutation of firms). We now establish that these equilibria constitute the entire equilibrium set.[28]

[28] More precisely, the proposition establishes essential uniqueness, up to the usual continuous-time degeneracies on sets of times of measure zero, as well as the off-path indeterminacies discussed in footnote 20.

Proposition 3. *There exist no perfect Bayesian equilibria, in pure or mixed strategies, beyond those characterized in Propositions 1 and 2.*

The bulk of the proof involves showing that, up to some technicalities, all equilibria must be in threshold strategies. The optimality of a threshold investing rule relies on an argument ruling out waiting for a (possibly random) period and then investing. Such a strategy would merely delay investment without conditioning it on the arrival of information in any useful way. So once it becomes optimal to wait at all, any optimal strategy must involve waiting until the other firm has invested. The optimality of a threshold prospecting rule is more technical, and requires studying the dynamics of the HJB equation. Essentially, the proof establishes that the moment shirking becomes even weakly optimal, a firm's value function must evolve in such a way that shirking remains strictly optimal forever afterward.

Within the class of equilibria in threshold strategies, the equilibrium set can be narrowed down by a straightforward classification argument. The symmetric equilibrium can be characterized as the unique equilibrium in which both firms stop investing on-path at the same time. Within this class, the only way that both firms can become passive at the same time in equilibrium is if both firms' beliefs reach π_A at this time. If some firm's terminal beliefs were any higher that firm would prefer to continue prospecting and investing afterward, and if its beliefs were any lower it would prefer to become passive sooner. Backward induction then pins down the symmetric equilibrium as the unique behavior consistent with this outcome.

The leader-follower equilibria can be characterized as the unique equilibria in the remaining case that some firm i remains active, that is invests along the equilibrium path, longer than the other. Let $\widehat{T}_{-i}$ be the time at which firm $-i$ becomes passive, and call firm $-i$ the follower. In this case firm i, the leader, is effectively in autarky after time $\widehat{T}_{-i}$ and works and invests immediately at all future times. To sustain an equilibrium, it must then be a best response to the leader's continuation strategy for the follower to stop investing on-path at time $\widehat{T}_{-i}$. This optimality condition uniquely pins down $\widehat{T}_{-i}$, which may be the time at which the follower either stops prospecting or stops investing depending on parameters. Once this time is pinned down, it can be shown that the leader's unique best response is to remain active prior to time $\widehat{T}_{-i}$, which uniquely determines the remainder of the equilibrium.

Due to the lengthy casework required to rule out all other equilibria, we have relegated the proof of this proposition to the online appendix.

5 Implications for venture capital

We have seen that our model has exactly two distinct equilibrium structures. In this section we analyze which structure yields higher total firm profits, and use the results to derive testable predictions about investor behavior in venture capital markets.

5.1 The profit-maximizing equilibrium

Let V^S be the expected payoff of each firm in the symmetric equilibrium, and V^L and V^F be the expected payoffs to the leader and follower, respectively, in the leader-follower equilibrium. Aggregate firm profits in the symmetric equilibrium are then $2V^S$, while in the leader-follower equilibrium they are $V^L + V^F$. The following proposition shows that the comparison between these two payoffs depends on the discount rate r.

Proposition 4. *If r is sufficiently small, then $V^L + V^F > 2V^S$. There exists a $\underline{c} < \overline{c}$ (independent of r) such that if $c > \underline{c}$, then $2V^S > V^L + V^F$ for r sufficiently large.*

This proposition establishes that when firm are patient, the leader-follower equilibrium yields higher total firm profits than the symmetric one, while when firms are impatient (and costs aren't too low) the symmetric equilibrium is superior. The basic tradeoff underpinning this result is that the symmetric equilibrium generates higher expected flow payoffs early in the project, while the leader-follower equilibrium generates higher payoffs late in the project. Consequently, the comparison between the two equilibria turns on the discount rate.

More precisely, note that in the leader-follower equilibrium some firm, say firm 2, becomes passive at time $\widehat{T}_2 = \min\{\overline{T}_2, T_2^*\} < T^A$, while in the symmetric equilibrium that firm would have continued actively prospecting and investing until time T^A. Thus firm 2 generates lower flow profits over the time interval $[\widehat{T}_2, T^A]$ in the leader-follower equilibrium as compared to the symmetric equilibrium.[29] On the other hand, firm 1 abandons the project at time T^A in the symmetric equilibrium, but remains active forever in the leader-follower equilibrium. That firm therefore generates lower flow profits over the time interval $[T^A, \infty)$ in the symmetric equilibrium compared to the leader-follower equilibrium.

When firms are patient, the long duration of firm 1's profit loss over the interval $[T^A, \infty)$ in the symmetric equilibrium dominates, and the leader-follower equilibrium yields higher total profits. Conversely, when firms are impatient, the earlier onset of firm 2's profit loss

[29]More precisely, total profits would increase if the follower remained active until some time slightly beyond $\widehat{T}_2$. It is not necessarily the case that profits would be maximized by remaining active all the way to T^A, as this activity implies a first-order loss of value for firm 2. However, the essence of the argument does not change by thinking about the inefficiency as being generated over the entire interval.

in the leader-follower equilibrium weighs heavily. The one subtlety to this argument is that the size of the interval $[\widehat{T}_1, T^A]$ shrinks as r grows. It turns out that nonetheless the profit loss generated is bounded away from 0 in the limit, and the size of the loss grows with c. Thus when c is large enough, the profit loss under the leader-follower equilibrium for large r exceeds that of the symmetric equilibrium. Note that the cost bound $\underline{c}$ need not be very stringent. In particular, in the proof of the proposition we show that $\underline{c} < 0$ when q^H and q^L are both sufficiently large.

The ambiguity of the total profit comparison comparison in our model stands in contrast to the findings of previous work on investment timing. A classic example is Gul and Lundholm (1995), who find that asymmetric equilibria eliminate the war of attrition inherent in symmetric play and reveal private information more quickly, improving aggregate welfare. By contrast, since information acquisition is endogenous in our model, asymmetric play generates an additional profit loss by reducing the incentives for the second mover to produce and reveal information. The relative performance of symmetric and asymmetric play then becomes a horse race between the war of attrition effect of the symmetric equilibrium, and the free-riding of the asymmetric equilibrium. This finding demonstrates the importance of modeling incentives for information acquisition alongside investment timing when both effects are present in applications.

5.2 Testable predictions

When applying our model to predict venture capitalist behavior in the market for entrepreneurial fundraising, the predictions of our model depend on how firms coordinate on an equilibrium. We focus on the possibility that firms choose the equilibrium which maximizes total profits across both firms. This selection is reasonable if firms interact simultaneously over multiple potential investments and may rotate their roles from project to project. The presence of such "multi-market contact" is born out by evidence on coinvestment networks among venture capitalists. Hochberg et al. (2007), for instance, find that "VCs repeatedly coinvest with a small set of other VCs" (pg. 265). In a setting with multiple simultaneous projects, the symmetry of the setting naturally suggests that equilibria should be chosen so that each firm makes the same total profits across all projects. Profits under this symmetry requirement are maximized by selecting the joint-profit-maximizing equilibrium for each project and rotating roles across projects, possibly excepting one project when the total number of projects is odd. In particular, when firms investigate a large number of markets, the joint-profit-maximizing equilibrium is selected in approximately all projects.[30]

As we just saw, the joint-profit-maximizing equilibrium changes with model parameters, in particular with the discount rate r. One way to interpret r is as the promised rate of return on funds raised by a venture capitalist from limited partners. This promised return is a function of a variety of external factors, including the competition for capital by venture firms. Our model therefore has testable implications for how investor behavior changes with market conditions.

When the promised return is high, our model predicts that investors will coordinate on symmetric play, with multiple investors simultaneously evaluating a startup and all investors abandoning the startup if an investment is not raised sufficiently quickly. Thus the mean evaluation time will be low, but the distribution of evaluation times will be right-truncated. Conversely, when the promised return on investment is low, investors will coordinate on a leader-follower structure, with eventually only one firm evaluating a given startup. In this case the distribution of evaluation times shifts outward compared to when the return on funds is high, and the distribution exhibits a right tail of long evaluation times without truncation.

As we discussed in Section 1.1, firms often adopt stratified roles in practice, with some firms leading evaluation and investment while other firms delay action until a leader has invested first. Our results suggest that this behavior may arise most frequently under market conditions in which venture capitalists' promised rate of return to limited partners is relatively low. Conversely, when the promised rate of return is high, strategic delays in action are counterproductive and less likely to arise.

6 Discussion of assumptions

Our model makes a number of simplifying assumptions to streamline our analysis and maintain tractability. We now briefly discuss these assumptions and how our results would be impacted by relaxing them.

Symmetry: We have assumed that both firms possess the same prospecting costs and acquire signals of equal precision. We expect that if firms were heterogeneous across either of these characteristics but heterogeneity were not too large, the equilibrium set would look qualitatively similar to the symmetric case, with two leader-follower equilibria and a

[30] Alternatively, this selection could be justified in a single-project model by including a public randomization device. Under public randomization, the optimal symmetric equilibrium would involve selection of the joint-profit-maximizing equilibrium structure and random assignment to each role.

"quasi-symmetric" equilibrium in which both firms eventually abandon the project.[31] The relative profitability of the leader-follower equilibrium with the efficient firm as the leader versus the symmetric equilibrium would also be naturally enhanced by heterogeneity. When heterogeneity is sufficiently large, we expect that the leader-follower equilibrium with the efficient firm as the leader would emerge as the unique equilibrium.[32]

Signal structure: We have assumed throughout our analysis was that each firm could acquire at most one signal about the project. So long as signals are sufficiently costly to acquire, this assumption is without loss, since then along the equilibrium path no firm would expend the effort to acquire multiple signals. In general, we expect the ability to acquire multiple signals would strengthen incentives for investment delay, as firms could flexibly acquire "just enough" information about the project, and then wait for an investment decision by another firm to provide the final piece of evidence convincing them to invest. Overall, we expect the interaction of free-riding and investment timing decisions to yield similar predictions in a model with a more complex information acquisition technology.

Number of firms: Our model focuses on a two-firm setting. Given appropriate assumptions on the incremental value of successive signals, we expect that analogs of our symmetric and leader-follower equilibria would continue to exist with many firms. Consider first the symmetric equilibrium. Our finding that each firm is incentivized to prospect when all firms drop out at the break-even threshold is not sensitive to the number of firms—from a given firm's perspective, the presence of more firms is equivalent to a higher prospecting rate, and we have shown that a symmetric equilibrium exists for any choice of $\overline{\lambda}$. As for the leader-follower equilibrium, a structure with one leader and many passive followers is clearly sustainable, as each successive follower finds it at least as profitable to remain passive as the ones before it. Beyond these equilibria, we conjecture that there may exist additional equilibria not present in the two-firm setting, potentially involving active prospecting and investment by a subset of firms and passivity by all other firms. Studying these equilibria could yield additional testable predictions on the optimal number of firms investigating a given project simultaneously.

[31] In general asymmetric firms would have different belief thresholds for abandonment, and so such an equilibrium would involve the more efficient firm quitting sooner so as to leave the less efficient firm at its single-player abandonment belief threshold. The less efficient firm would then continue prospecting (which it is weakly willing to do) for a time period calibrated to induce the efficient firm's quitting.

[32] See also Awaya and Krishna (2019) for a related model of patent races in which the more-informed firm always takes an irreversible public action first in equilibrium when the gap in information is large.

7 Conclusion

We study a model of strategic investment timing with endogenous information acquisition, with the aim of understanding the interplay of incentives for free-riding and investment delay present in settings such as the market for entrepreneurial fundraising. We find that the symmetric equilibrium exhibits no free-riding or investment delay, but that both firms eventually abandon the project if neither firm has invested. Meanwhile the asymmetric equilibria exhibit free-riding, investment delay, or both, with the mix determined by the costs of prospecting, tilting toward free-riding as costs rise.

We find that either equilibrium can yield higher firm profits depending on model parameters, in particular the discount rate. This stands in contrast to existing models of investment timing, where the primary inefficiency is a war of attrition which is alleviated by asymmetric play. When firms investigate many projects simultaneously, the symmetry of our setting suggests that firms may coordinate on the joint-profit-maximizing equilibrium for each project. The dependence of this equilibrium on the discount rate therefore yields testable predictions for venture capital behavior across varying market conditions.

A central feature of our model is the nonrivalry of investment. It would be interesting to extend our model to incorporate investment externalities. In one direction, significant payoff rivalries may eliminate incentives for free-riding and delay and lead to a unique equilibrium in which both firms evaluate the project intensively and then abandon it. In practice entrepreneurs may have some control over investment rivalry, for instance by offering better investment terms to early investors. Extending our model to include rivalry would permit analysis of optimal design of deal terms. In the other direction, some projects may feature increasing returns to scale as more capital is raised. In practice in such environments, venture capitalists often form syndicates which ensure that sufficient capital is raised before any individual investor commits to investing. Extending our model to include returns to scale could yield insights into when such syndication is most useful and effective.

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Appendices

A The HJB equation

In this appendix we characterize the HJB equation determining each firm's optimal prospecting rule given a regular strategy by the other firm involving non-random prospecting and a threshold investment rule.

Let $\overline{V}$ be i's continuation value upon seeing firm $-i$ invest. Given that $-i$ uses a regular strategy, i's posterior beliefs from this point onward are fixed at π_+ so long as i has not acquired a signal. As in the single-player problem, i's optimal continuation strategy in this contingency is either to invest immediately, or to acquire an additional signal and invest iff that signal is High. Therefore

$$\overline{V} = \max\left\{\pi_+ R - 1, \frac{\overline{\lambda}}{\overline{\lambda} + r}(h(\pi_+)(\pi_{++}R - 1) - c)\right\}.$$

Recall from Lemma 1 that r^* is defined as the minimal discount rate at which firm i invests immediately after seeing firm $-i$ invest, i.e. the first term on the rhs dominates. Some aspects of the equilibrium analysis will depend on which of these two values $\overline{V}$ takes. For convenience, we will use the following terminology:

Definition A.1. *If $r \leq r^*$, then signals are* complements*. Otherwise, signals are* substitutes*.*

Let $V^i(t)$ be firm i's equilibrium continuation value function conditional on receiving no signal and seeing no investment by firm $-i$ up to time t. Let T^*_{-i} be the time at which firm $-i$ begins waiting upon receiving a High signal. For all times $t \geq T^*_{-i}$, the firm's beliefs are fixed at $\mu^i(T^*_{-i})$, and $V^i(t)$ is the value of the corresponding single-player problem. So consider times $t < T^*_{-i}$. By standard arguments, V^i is an absolutely continuous function satisfying the HJB equation

$$\begin{aligned} rV^i(t) = &\max_{\lambda \in [0,\overline{\lambda}]} \left\{\lambda\left(\widetilde{V}^i(t) - c - V^i(t)\right)\right\} \\ &+ \Pr(s_t^{-i} = \emptyset \mid \tau^{-i}(\emptyset) \geq t)\lambda^{-i}(t)h(\pi_0)(\overline{V} - V^i(t)) + \dot{V}^i(t). \end{aligned}$$

for almost all times $t < T^*_{-i}$, where $\widetilde{V}^i(t)$ is firm i's continuation value after having received a signal at time t.

In Lemma O.8 of the online appendix, we explicitly characterize $\Pr(s_t^{-i} = \emptyset \mid \tau^{-i}(\emptyset) \geq t)$ in terms of $\lambda^{-i}(t)$, $\mu^i(t)$, and $\dot{\mu}^i(t)$. Using this expression, the HJB equation may be written in the following compact form, which will be regularly invoked in proofs:

$$rV^i(t) = \overline{\lambda}\left(\widetilde{V}^i(t) - c - V^i(t)\right)_+ - \frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)}(\overline{V} - V^i(t)) + \dot{V}^i(t).$$

Whenever firm i optimally invests immediately following observation of a High signal, we have $\widetilde{V}^i(t) = h(\mu^i(t))(\mu^i_+(t)R - 1)$. Lemma O.2 in the online appendix derives the useful associated identity

$$h(\mu)(\mu_+ R - 1) - c = K(\mu - \pi_A),$$

where $\mu_+ = q^H \mu / h(\mu)$ and $K \equiv q^H(R-1) + (1-q^L)$. This identity will often be invoked when applying the HJB equation in proofs.

B Proofs

B.1 Proof of Proposition 1

Fix a firm i, and consider any continuation game in which it has already obtained a High signal. Because $\mu^i \geq \pi_A$, therefore $\mu^i_+(t) > 1/R$ for all time. So investment is always profitable at each future time, regardless of whether the other firm has invested or not. Therefore the payoff of any investment strategy which occasionally never invests is dominated by the payoff of an investment strategy which always eventually invests, and due to time discounting all strategies involving delay in investment yield a strictly lower payoff than a strategy which invests immediately. So investing immediately is an optimal continuation strategy in all such continuation games, implying optimality of $T_i^* = \infty$ for each firm.

Now consider firm i's optimal prospecting problem prior to obtaining a signal. Subsequent to the cutoff time T^A its beliefs are exactly π_A, so no prospecting is trivially an optimal strategy at this point. So consider times prior to T^A. We first show that $V^\dagger(t) = K(\mu^i(t) - \pi_A)$ is a supersolution to firm i's HJB equation on $[0, T^A]$. Recall from Appendix A that the HJB equation for firm i in this regime may be written $F(V^i(t), t) = 0$, where

$$F(w,t) \equiv rw(t) - \overline{\lambda}\left(K(\mu^{\overline{\lambda}}(t) - \pi_A) - w(t)\right)_+ + \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}\left(\overline{V} - w(t)\right) - \dot{w}(t).$$

We will show that $F(V^\dagger, t) \geq 0$ for all $t \leq T^A$. Inserting $V^\dagger$ into the definition of F and combining terms shows that

$$F(V^\dagger, t) = rV^\dagger(t) + \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}(\overline{V} - K(\pi_+ - \pi_A)).$$

Now that for $t \leq T^A$, $V^\dagger(t) \geq 0$ and $\frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)} < 0$. Lemma O.3 in the online appendix establishes the bound $\overline{V} \leq K(\pi_+ - \pi_A)$. So $F(V^\dagger, t) \geq 0$, i.e. $V^\dagger$ is a supersolution to the HJB equation on $[0, T^A]$.

Now, note that $V^\dagger(T^A) = 0$ by definition of T^A, while also $V^i(T^A) = 0$ given that firm i is in autarky with beliefs π_A subsequent to T^A. Therefore $V^\dagger(T^A) = V^i(T^A)$, and since V^i satisfies the HJB equation while $V^\dagger$ is a supersolution on $[0, T^A]$, it must be that $V^\dagger(t) \geq V^i(t)$ for all $t \in [0, T^A]$. The HJB equation then implies that prospecting at the maximum rate prior to T^A is an optimal strategy.

As both the prospecting and investment strategy of each firm under the specified strategy profile are best responses to the other firm's strategy, the strategy profile constitutes a perfect Bayesian equilibrium.

B.2 Proof of Proposition 2

We first characterize the follower's best response to the leader. To this end, we first define a pair of belief thresholds, which will turn out to pin down the times at which the follower stops prospecting and stops investing. Suppose that firm i has current posterior beliefs $\mu \in [\pi_-, \pi_0]$ about the state, following a history in which it has no signal and has not seen firm $-i$ invest. Further suppose firm $-i$ employs the leader strategy. If firm i then receives a High signal, let $\Delta(\mu)$ be the difference in continuation payoffs between waiting for firm $-i$ to invest, then investing immediately afterward, versus investing immediately.

The following lemma establishes that the value of waiting rises relative to the value of investing immediately as beliefs drop, and that eventually waiting dominates immediate investing. (The proof of this and all other auxiliary lemmas used in the proof of this proposition can be found in the online appendix.)

Lemma B.1. Δ *is a strictly decreasing function of* μ*, and* $\Delta(\pi_-) > 0$*. Also,*

$$\Delta(\pi_0) = \frac{\overline{\lambda}}{\overline{\lambda} + r} h(\pi_+)(\pi_{++}R - 1) - (\pi_+ R - 1).$$

In particular, $\Delta(\pi_0) > 0$ *whenever signals are complements.*

Now, define a belief threshold $\mu^* \in (\pi_-, \pi_0]$ to be the belief at which the continuation payoffs of investing and waiting are equalized:

$$\mu^* \equiv \begin{cases} \pi_0, & \Delta(\pi_0) \geq 0, \\ \Delta^{-1}(0), & \Delta(\pi_0) < 0. \end{cases}$$

When mapped onto a corresponding time at which these beliefs are reached, μ^* will pin down the time at which the follower stops investing in equilibrium.

Next, suppose that firm i follows a strategy involving no prospecting prior to observing the leader investing, while firm $-i$ employs the leader strategy. Suppose firm i has current posterior beliefs $\mu \in [\pi_-, \pi_0]$ about the state, following a history in which it has no signal and has not seen firm $-i$ invest. Under these conditions, let $\widetilde{\Delta}(\mu)$ be the negative of the marginal change in firm i's continuation value from temporarily prospecting.

Lemma B.2. *$\widetilde{\Delta}$ is a strictly decreasing function and $\widetilde{\Delta}(\pi_-) > 0$.*

In light of the previous lemma, define a belief threshold $\overline{\mu} \in (\pi_-, \pi_0]$ by

$$\overline{\mu} \equiv \begin{cases} \pi_0, & \widetilde{\Delta}(\pi_0) \geq 0, \\ \widetilde{\Delta}^{-1}(0), & \widetilde{\Delta}(\pi_0) < 0. \end{cases}$$

This threshold will pin down the time at which the follower stops prospecting.

The following lemma characterizes the follower's unique best response to the leader's strategy, by showing that the follower prospects until its beliefs hit $\overline{\mu}$, then shirks; and invests immediately until its beliefs hit μ^*, then waits. Note that as $\overline{\mu}, \mu^* > \pi_-$, both thresholds are reached in finite time.

Lemma B.3. *Suppose some firm $-i$ chooses the threshold strategy $T^*_{-i} = \overline{T}_{-i} = \infty$. Then firm i's unique best response is the threshold strategy characterized by $T^*_i = (\mu^{\overline{\lambda}})^{-1}(\mu^*)$ and $\overline{T}_i = (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$.*

We next establish that when c is sufficiently small, the follower prospects for a period of time after it begins waiting to invest.

Lemma B.4. *$\overline{\mu} < \min\{\mu^*, \pi_A\}$ when c is sufficiently small.*

In the remainder of the proof, we establish that the leader's strategy is a best reply to the follower strategy characterized in Lemma B.3. We first establish the result under an auxiliary condition on beliefs.

Lemma B.5. *Suppose that some firm i employs a threshold strategy satisfying $\mu^{\overline{\lambda}}(\min\{T^*_i, \overline{T}_i\}) > \pi^A$. Then firm $-i$'s unique best reply is the threshold strategy $T^*_{-i} = \overline{T}_{-i} = \infty$.*

The following lemma completes the equilibrium verification by showing that the follower's strategy satisfies the auxiliary belief condition of Lemma B.5.

Lemma B.6. $\max\{\overline{\mu}, \mu^*\} > \pi_A$.

Finally, we establish properties of μ^* and $\overline{\mu}$ used in the proof of Lemma 2.

Lemma B.7. *If r is sufficiently small, then $\overline{\mu} = \pi_0$ for costs sufficiently close to $\overline{c}$.*

Lemma B.8. *$\overline{\mu}$ is increasing in c, and strictly increasing whenever $\overline{\mu} < \pi_0$.*

B.3 Proof of Lemma 2

Lemma B.8 implies that $\overline{T}_{-i}$ is decreasing in c, and is strictly decreasing whenever it is positive. Meanwhile T^*_{-i} is independent of c, because the function Δ characterizing μ^* does not involve c. Then either there exists a $c^* < \bar{c}$ below which $\overline{T}_{-i} > T^*_{-i}$ and above which $\overline{T}_{-i} \leq T^*_{-i}$ (with the inequality possibly weak if $T^*_{-i} = 0$) or else $\overline{T}_{-i} > T^*_{-i}$ for every $c \leq \bar{c}$. Letting $c^* = \bar{c}$ in the latter case establishes existence of the c^* threshold claimed in the first part of the lemma. Further, Lemma B.7 establishes that when r is sufficiently small, $\overline{T}_{-i} = 0$ for c sufficiently close to $\bar{c}$. Thus for r sufficiently small $\overline{T}_{-i} \leq T^*_{-i}$ when c is close to $\bar{c}$, meaning that $c^* < \bar{c}$. This establishes the first part of the lemma.

As for the second part, the proof of Lemma B.4 establishes $\widetilde{\Delta}(\pi_A) < 0$ for c sufficiently small. We will additionally prove that $\widetilde{\Delta}(\pi_A) > 0$ for c sufficiently close to $\bar{c}$. Since $\widetilde{\Delta}(\mu)$ is strictly decreasing in μ, this implies that $\overline{\mu} < \pi_A$ for c small and $\overline{\mu} > \pi_A$ for c close to $\bar{c}$, i.e. $\overline{T}_{-i} > T^A$ for c small and $\overline{T}_{-i} < T^A$ for c close to $\bar{c}$. Thus there exist cost thresholds $c_* \geq c'_*$ in $(0, \bar{c})$ satisfying the conditions of the lemma. If $c^* = \bar{c}$, then automatically $c_* < c^*$. So suppose instead that $c^* < \bar{c}$. Recall by Lemma B.6 that $\min\{T^*_{-i}, \overline{T}_{-i}\} < T^A$ for any cost level. So consider any cost $c \geq c^*$, for which $\overline{T}_{-i} \leq T^*_{-i}$. Then $\min\{T^*_{-i}, \overline{T}_{-i}\} < T^A$ requires that $\overline{T}_{-i} < T^A$ for such a cost level, so that by continuity of $\overline{T}_{-i} - T^A$ the threshold c_* may always be taken to be strictly less than $\bar{c}$, as desired. Finally, note that whenever $\widetilde{\Delta}(\pi_A)$ satisfies single crossing in c, we may additionally take $c_* = c'_*$.

It remains to establish the desired sign of $\widetilde{\Delta}(\pi_A)$ for large c, as well as sufficient conditions under which $\widetilde{\Delta}(\pi_A)$ satisfies single-crossing. $\widetilde{\Delta}$ is explicitly characterized in Lemma O.18 of the online appendix. Evaluated at π_A, the expression is

$$\widetilde{\Delta}(\pi_A) = \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} - \max\left\{h(\pi_A)(\pi_{A+}R - 1), \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}h(\pi_+)(\pi_{++}R - 1)\right\} + c.$$

If $\pi_A \geq \mu^*$, then the first term in the max dominates and

$$\widetilde{\Delta}(\pi_A) = \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} - h(\pi_A)(\pi_{A+}R - 1) + c = \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V},$$

where $h(\pi_A)(\pi_{A+}R - 1) = c$ by definition of π_A. Recall that $\overline{V} > 0$ and $\pi_{+-} < 1/R$, so that $\pi_A > \pi_-$ for any $c \geq 0$. Thus $\widetilde{\Delta}(\pi_A) > 0$ whenever $\pi_A \geq \mu^*$. Since $\widetilde{\Delta}(\pi_A) < 0$ for c sufficiently small, this result implies in particular that $\pi_A < \mu^*$ for c sufficiently small.

On the other hand, if $\pi_A < \mu^*$ then the second term in the max dominates and

$$\widetilde{\Delta}(\pi_A) = c - \frac{\pi_A - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r} \left(h(\pi_+)(\pi_{++}R - 1) - \overline{V} \right).$$

With these formulae in hand, we analyze how $\widetilde{\Delta}(\pi_A)$ varies with c. Note that $\widetilde{\Delta}(\pi_A)$ varies with c both through its explicit dependence, as well as through the dependence of π_A and $\overline{V}$ on c. The autarky threshold π_A is a strictly increasing affine function of c, while $\overline{V}$ is weakly decreasing, and strictly decreasing whenever signals are complements. Finally, recall that μ^* is independent of c.

Suppose first that $\mu^* < \pi_0$, i.e. $\Delta(\pi_0) < 0$. By Lemma B.1 this is equivalent to the condition $\frac{\overline{\lambda}}{\overline{\lambda}+r} h(\pi_+)(\pi_{++}R - 1) < \pi_+ R - 1$. Note that this condition holds whenever r is sufficiently large. In this regime $\overline{V} = \pi_+ R - 1$ for any $c \geq 0$, so that

$$\widetilde{\Delta}(\pi_A) = \overline{\chi}(c) \equiv c - \frac{\pi_A - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r} \overline{c}$$

whenever $\pi_A < \mu^*$. Since π_A is affine in c, $\overline{\chi}$ is an affine function of c. Evaluated at $c = 0$ it is strictly negative given that $\pi_A > \pi_-$. Meanwhile evaluated at $c = \overline{c}$ it is strictly positive given that $\pi_A < \pi_0$. So $\overline{\chi}$ is a strictly increasing function, meaning that $\widetilde{\Delta}(\pi_A)$ is strictly increasing in c for costs such that $\pi_A < \mu^*$. There are then two possibilities: either $\pi_A < \mu^*$ for all $c \leq \overline{c}$, or else $\pi_A \geq \mu^*$ for c sufficiently large. (Recall that π_A is strictly increasing in c while μ^* is independent of c, and $\pi_A < \mu^*$ for c sufficiently small.) In the first case we have seen that $\widetilde{\Delta}(\pi_A)$ is a strictly increasing, continuous function of c which is strictly negative at $c = 0$ and strictly positive at $c = \overline{c}$, meaning there exists a unique $c_* \in (0, \overline{c})$ at which it vanishes. In the second case, since $\widetilde{\Delta}(\pi_A)$ is a continuous function, is strictly increasing when $\pi_A < \mu^*$, is strictly positive when $\pi_A \geq \mu^*$, and is strictly negative at 0, there again exists a unique $c_* \in (0, \overline{c})$ at which it vanishes.

Now suppose that $\mu^* = \pi_0$, i.e. $\Delta(\pi_0) \geq 0$. This condition holds whenever r is sufficiently small. In this regime trivially $\pi_A < \mu^*$ for all c given that $\pi_A < \pi_0$. Further $\frac{\overline{\lambda}}{\overline{\lambda}+r} h(\pi_+)(\pi_{++}R - 1) \geq \pi_+ R - 1$, and so there exists a unique $\underline{c} \in [0, c^*)$ such that

$$\frac{\overline{\lambda}}{\overline{\lambda} + r} \left(h(\pi_+)(\pi_{++}R - 1) - \underline{c} \right) = \pi_+ R - 1.$$

For $c < \underline{c}$ we have $\overline{V} = \frac{\overline{\lambda}}{\overline{\lambda}+r} \left(h(\pi_+)(\pi_{++}R - 1) - c \right)$, while for $c \geq \underline{c}$ we have $\overline{V} = \pi_+ R - 1$.

Define

$$\underline{\chi}(c) \equiv c - \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}\left(h(\pi_+)(\pi_{++}R-1) - \frac{\overline{\lambda}}{\overline{\lambda}+r}\left(h(\pi_+)(\pi_{++}R-1)-c\right)\right).$$

Then

$$\widetilde{\Delta}(\pi_A) = \begin{cases} \underline{\chi}(c), & c < \underline{c} \\ \overline{\chi}(c), & c \geq \underline{c}. \end{cases}$$

Note that $\underline{\chi}$ is a concave quadratic in c satisfying

$$\underline{\chi}(0) = -\frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}\left(1 - \frac{\overline{\lambda}}{\overline{\lambda}+r}\right)h(\pi_+)(\pi_{++}R-1) < 0,$$

while as established earlier $\overline{\chi}$ is a strictly increasing function of c satisfying $\overline{\chi}(0) < 0$ and $\overline{\chi}(\overline{c}) > 0$. Thus given $\underline{c} < c^*$ we have $\widetilde{\Delta}(\pi_A) > 0$ for c sufficiently close to c^*, as desired.

Note that in general single-crossing of $\widetilde{\Delta}(\pi_A)$ is not ensured, as it is possible in principle that $\underline{\chi}(c)$ crosses 0 twice on the interval $[0, \underline{c}]$. A sufficient condition for single-crossing is that $\underline{\chi}(\overline{c}) > 0$. This quantity may be written

$$\underline{\chi}(\overline{c}) = \overline{c} - \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}\left(h(\pi_+)(\pi_{++}R-1) - \frac{\overline{\lambda}}{\overline{\lambda}+r}(\pi_+R-1)\right).$$

At $r = 0$ this expression reduces to

$$\underline{\chi}(\overline{c}) = \overline{c} - \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\overline{c} > 0,$$

so $\underline{\chi}(\overline{c}) > 0$ for r sufficiently small. Also, note that $\underline{\chi}(\overline{c})$ is a convex quadratic in $\overline{\lambda}/(\overline{\lambda}+r)$, which is minimized at

$$\frac{\overline{\lambda}}{\overline{\lambda}+r} = \frac{h(\pi_+)(\pi_{++}R-1)}{2(\pi_+R-1)}.$$

Whenever this expression is at least 1, then given the constraint $\frac{\overline{\lambda}}{\overline{\lambda}+r} \leq 1$ the quantity $\underline{\chi}(\overline{c})$ is minimized at $\frac{\overline{\lambda}}{\overline{\lambda}+r} = 1$, i.e. $r = 0$, in which case as we've seen $\underline{\chi}(\overline{c}) > 0$. Thus a sufficient condition for $\underline{\chi}(\overline{c}) > 0$ for any value of r is that $h(\pi_+)(\pi_{++}R-1) \geq 2(\pi_+R-1)$.

B.4 Proof of Proposition 4

We first analyze welfare when signals are complements, and then consider the case of substitutes. Consider first the symmetric equilibrium. In this equilibrium, one best response for each firm is the equilibrium strategy of prospecting at the maximum rate until time T^A and then abandoning the project. However, for $t \geq T^A$ each firm is in autarky with beliefs fixed at π_A. Hence another best response subsequent to time T^A is to continue prospecting at the maximum rate forever and to invest if a High signal is received. Hence there exists a best response of the following sort - as long as there is no investment by the other firm, prospect forever until receiving a signal; if the other firm invests, continue prospecting forever until receiving a signal; and when a signal is received, invest immediately iff it is High, otherwise never invest. But this strategy is exactly the optimal strategy under autarky. And as the presence of the other firm brings informational externalities but no payoff externalities, this strategy must yield the autarky payoff. Hence each firm's equilibrium payoff must be the same as the autarky payoff: $V^S = V^A$, where we let V^A denote the payoff in the one-player problem.

In the leader-follower equilibrium, the follower never invests before the leader. Thus the leader is effectively in autarky and receives his autarky payoff: $V^L = V^A$. Meanwhile, suppose the leader plays its equilibrium strategy. By Lemma B.3 and the fact that $T^* = 0$ under complementary signals, it is a unique optimal continuation strategy for the follower to wait forever upon receiving a High signal at any time. Note that this is true regardless of the follower's prospecting strategy. So consider the payoff to the follower of employing a modified autarky strategy which preserves the autarky prospecting rule but waits forever rather than investing immediately after obtaining a High signal. This modification must strictly improve on the payoff of playing the autarky strategy, given the positive probability of obtaining a signal under this strategy and the unique optimality of waiting upon receiving a signal at any point in time. And the follower's equilibrium payoff must be at least as high as this modified autarky strategy, so in equilibrium the follower earns strictly more than by playing the autarky strategy. Meanwhile, playing the autarky strategy yields V^A regardless of the leader's strategy, so $V^F > V^A$. Combining the results of the previous two paragraphs, we find that $V^L + V^F > 2V^A = 2V^S$.

Now suppose signals are substitutes. Note that the symmetric and leader's equilibrium strategy are independent of r, and thus expected discounted profits V^L and V^S are immediately continuous in r. Meanwhile the follower's equilibrium strategy maximizes expected discounted profits over two scalar time thresholds at which prospecting and investing cease. Then as the expected profit function is continuous in r for any choice of thresholds, by the theorem of the maximum V^S is continuous in r as well. Now note that $V^L + V^F > 2V^S$ for the threshold discount rate r^* at which

$$\frac{\overline{\lambda}}{\overline{\lambda}+r^*}(h(\pi_+)(\pi_{++}R-1)-c)=\pi_+R-1,$$

i.e. the largest r under which signals are complementary. Continuity therefore implies that $V^L+V^F>2V^S$ for $r>r^*$ sufficiently small.

We now consider the limit of large r. We proceed by explicitly calculating each value as a sum of expected discounted flow profits over each moment of time in which a firm has not received a signal or seen the other firm invest. The flow of profits accounts for both any effort expended in that instant, as well as the probability of arrival of a signal or another investment, which each yield an additional flow of expected profits. In the symmetric equilibrium, these profits are just

$$rV^S=\int_0^{T^A} dt\, re^{-rt}e^{-\overline{\lambda}t}\left(1-\left(1-e^{-\overline{\lambda}t}\right)h(\pi_0)\right)(\pi_1(t)+\pi_2(t)),$$

where

$$\pi_1(t)\equiv\overline{\lambda}\left(h(\mu^{\overline{\lambda}}(t))\left(\mu_+^{\overline{\lambda}}(t)R-1\right)-c\right)$$

and

$$\pi_2(t)\equiv-\frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+-\mu^{\overline{\lambda}}(t)}(\pi_+R-1).$$

The terms $e^{-rt}e^{-\overline{\lambda}t}\left(1-\left(1-e^{-\overline{\lambda}t}\right)h(\pi_0)\right)$ discount for time and the probability that the firm has not yet received a signal or seen investment by the other firm. Meanwhile the term $\pi_1(t)+\pi_2(t)$ capture the firm's expected flow profits in this information set; $\pi_1(t)$ accounts for effort costs and the arrival of a signal, while $\pi_2(t)$ accounts for arrival of an investment by the other firm. The derivation of these flow profit representations is as in the HJB equation of Appendix A.

To calculate leader and follower flow profits, we first show that when r is large, $\overline{T}<T^*$. To see this, note that $\overline{T}=(\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$ and $T^*=(\mu^{\overline{\lambda}})^{-1}(\mu^*)$, where for large r $\overline{\mu}$ solves

$$\frac{\mu-\pi_-}{\pi_+-\pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}(\pi_+R-1)=h(\mu)(\mu_+R-1)-c,$$

while μ^* solves

$$\frac{\mu_+-\pi_{+-}}{\pi_{++}-\pi_{+-}}\frac{\overline{\lambda}}{\overline{\lambda}+r}(\pi_{++}R-1)=\mu_+R-1.$$

Thus for large r, $\overline{\mu}$ is close to the solution to $h(\mu)(\mu_+R-1)-c=0$, i.e. π_A, while μ^* is close to the solution to $\mu_+ R - 1 = 0$. This implies in particular that $\overline{\mu} > \mu^*$, i.e. $\overline{T} < T^*$. Note also that $\overline{\mu} > \pi_A$, so $\overline{T} < T^A$ for large r.

Using this result, for large r expected profits for the leader may be written

$$rV^L = \int_0^\infty dt\, re^{-rt} e^{-\overline{\lambda}t} \left(1 - \left(1 - e^{-\overline{\lambda}\min\{t,\overline{T}\}}\right) h(\pi_0)\right) \left(\pi_1(\min\{t,\overline{T}\}) + \mathbf{1}\{t < \overline{T}\}\pi_2(t)\right).$$

An equivalent, more convenient representation of this expression is obtained by explicitly evaluating the integral for times $t \geq \overline{T}$, yielding

$$\begin{aligned} rV^L = &\int_0^{\overline{T}} dt\, re^{-rt} e^{-\overline{\lambda}t} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) (\pi_1(t) + \pi_2(t)) \\ &+ \frac{r}{\overline{\lambda} + r} e^{-r\overline{T}} e^{-\overline{\lambda}\,\overline{T}} \left(1 - \left(1 - e^{-\overline{\lambda}\,\overline{T}}\right) h(\pi_0)\right) \pi_1(\overline{T}). \end{aligned}$$

Finally, in the same large-r regime follower profits are

$$rV^F = \int_0^\infty dt\, re^{-rt} e^{-\overline{\lambda}\min\{t,\overline{T}\}} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) \left(\mathbf{1}\{t < \overline{T}\}\pi_1(t) + \pi_2(t)\right).$$

We now compute the difference $r(2V^S - V^L - V^F)$. Note that each of V^S, V^L, V^F contains an identical integral over the range $[0, \overline{T}]$, so these terms cancel out in the sum. Each remaining term contains a common overall discount of $e^{-r\overline{T}}$, which does not affect the sign of the sum and may be factored out. What remains is

$$\begin{aligned} re^{r\overline{T}}(2V^S - V^L - V^F) = 2&\int_{\overline{T}}^{T^A} dt\, re^{-r(t-\overline{T})} e^{-\overline{\lambda}t} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) (\pi_1(t) + \pi_2(t)) \\ &- \frac{r}{\overline{\lambda} + r} e^{-\overline{\lambda}\,\overline{T}} \left(1 - \left(1 - e^{-\overline{\lambda}\,\overline{T}}\right) h(\pi_0)\right) \pi_1(\overline{T}) \\ &- \int_{\overline{T}}^\infty dt\, re^{-r(t-\overline{T})} e^{-\overline{\lambda}\,\overline{T}} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) \pi_2(t). \end{aligned}$$

We now evaluate the limit of this expression as $r \to \infty$. First, note that $\overline{T} \to T^A$ as $r \to \infty$, and $\pi_1(T^A) = 0$ by definition of T^A. Thus the second term vanishes in the limit. To evaluate the third integral, make the substitution $t' = r(t - \overline{T})$. As π_2 is bounded, the resulting integrand is uniformly bounded for all t' and r. Then by the bounded convergence theorem,

$$\lim_{r\to\infty} \int_{\overline{T}}^\infty dt\, re^{-r(t-\overline{T})} e^{-\overline{\lambda}\,\overline{T}} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) \pi_2(t) = e^{-\overline{\lambda}\,\overline{T}} \left(1 - \left(1 - e^{-\overline{\lambda}T^A}\right) h(\pi_0)\right) \pi_2(T^A).$$

To the evaluate the remaining integral, we invoke the following lemma, whose proof can be found in the online appendix.

Lemma B.9. $\lim_{r\to\infty} r(T^A - \overline{T}) = (\pi_+ R - 1)/(h(\pi_+)(\pi_{++}R - 1) - c)$.

In the remaining integral, make the substitution $t' = r(t - \overline{T})$. Recall that π_1 and π_2 are both bounded functions, so the resulting integrand is uniformly bounded. Additionally, the definition of T^A implies that $\pi_1(T^A) = 0$. Hence the bounded convergence theorem yields the result

$$\begin{aligned}
&\lim_{r\to\infty} \int_{\overline{T}}^{T^A} dt\, re^{-r(t-\overline{T})} e^{-\overline{\lambda}t} \left(1 - \left(1 - e^{-\overline{\lambda}t}\right) h(\pi_0)\right) (\pi_1(t) + \pi_2(t)) \\
&= \left(1 - \exp\left(-\frac{\pi_+ R - 1}{h(\pi_+)(\pi_{++}R - 1) - c}\right)\right) e^{-\overline{\lambda}T^A} \left(1 - \left(1 - e^{-\overline{\lambda}T^A}\right) h(\pi_0)\right) \pi_2(T^A).
\end{aligned}$$

Combining these calculations yields

$$\begin{aligned}
&\lim_{r\to\infty} r(2V^S - V^L - V^F) \\
&= \left(1 - 2\exp\left(-\frac{\pi_+ R - 1}{h(\pi_+)(\pi_{++}R - 1) - c}\right)\right) e^{-\overline{\lambda}T^A} \left(1 - \left(1 - e^{-\overline{\lambda}T^A}\right) h(\pi_0)\right) \pi_2(T^A).
\end{aligned}$$

Since $\pi_2(T^A) > 0$, the sign of $2V^S - V^L - V^F$ for large r is therefore the same as the sign of $1 - 2\exp(-(\pi_+ R - 1)/(h(\pi_+)(\pi_{++}R - 1) - c))$. In particular, $2V^S > V^L + V^F$ for large r whenever

$$\frac{\pi_+ R - 1}{h(\pi_+)(\pi_{++}R - 1) - c} > \log 2.$$

Letting

$$\underline{c} \equiv h(\pi_+)(\pi_{++}R - 1) - \frac{1}{\log 2}(\pi_+ R - 1),$$

this condition is equivalent to $c > \underline{c}$, given the additional constraint that $c \leq \overline{c}$ by Assumption 3. Note that $\underline{c}$ is independent of r and $\underline{c} < \overline{c}$ given that $\log 2 < 1$, as claimed in the proposition statement.

Finally, we derive conditions under which $\underline{c} < 0$, as referenced in the body of the paper. Suppose that $q^H = q^L = q$. Note that Assumptions 1 and 2 are both automatically satisfied when $q^H = q^L$. Further, π_A is decreasing in q, and so if Assumption 4 is satisfied for some q, it is satisfied for all larger q. Observe that as q approaches 1, π_+, π_{++}, and $h(\pi_{++})$ all approach 1. Thus in this limit $h(\pi_+)(\pi_{++}R - 1)$ approaches $\pi_+ R - 1$, in which case $\underline{c} < 0$. And given that $\underline{c} < \overline{c}$, for any q large enough that $\underline{c} < 0$, there exist a range of costs satisfying Assumption 3. These arguments also hold if q^H and q^L are sufficiently similar and large.

Material intended for online appendix

O.1 Regular strategies

In this appendix we establish that in any perfect Bayesian equilibrium, lack of investment is (weakly) bad news about the state, while investment indicates that the other firm has received a High signal.

Definition O.1. *A firm's strategy is* regular *if:*

- *Investment never occurs after receipt of a Low signal,*
- *Investment without a signal occurs only in histories in which the other firm has invested.*

Lemma O.1. *In any perfect Bayesian equilibrium, each firm's strategy is regular.*

Proof. First consider a firm who has received a Low signal. Then regardless of his beliefs about the content of any signal received by the other firm, his posterior belief that the state is Good cannot be higher than π_0. As $\pi_0 R - 1 < 0$ by assumption, investing is never optimal at any point in the future.

To establish the second property of regularity, we first show that in equilibrium firms are always able to use Bayes' rule to update their beliefs about the state no matter the history of the game. Fix an equilibrium σ, and suppose by way of contradiction that at some time t^* and following some history, 1) neither firm has invested by time t^*; 2) Bayes' rule applies for both firms at all $t < t^*$, but at t^* firm i cannot use Bayes' rule to form a posterior probability about θ. These two conditions imply that according to σ, firm $-i$ should have invested with probability 1 prior to t^* absent any investment by firm i. (Otherwise Bayes' rule would still be applicable at time t^*.) We already know that in any PBE, firm $-i$ will never invest if in possession of a Low signal. Therefore if firm $-i$ had a Low signal with some probability by time t^*, Bayes' rule would still apply for firm i. Hence firm $-i$ cannot have obtained a signal, hence cannot have prospected prior to t^*. In this case any updating to firm $-i$'s beliefs prior to t^* must come solely from social learning due to the absence of firm i's investment.

Bayes' rule applies for firm $-i$ at all $t < t^*$, meaning that with some probability under σ^i, firm i did not invest prior to t^*. And we already know that in any PBE, firm i never invests when in possession of a Low signal. Thus no matter how frequently it would invest when in possession of no signal or a High signal, the inference $-i$ must make about the state from lack of investment is weakly negative. Hence firm $-i$'s beliefs at all times prior to t^* must be no higher than π_0, meaning the payoff from investing is no higher than $R\pi_0 - 1 < 0$. This contradicts the assumption that σ is an equilibrium.

So under any equilibrium, if neither firm has invested by time t^*, and Bayes' rule applies for both firms at all prior times, then each firm must be able to use Bayes' rule to form beliefs at time t^*. Since this reasoning applies for every t^* and every history, it must be that Bayes' rule applies for both firms at all times following all histories in which neither firm has invested. It then follows from the argument of the previous two paragraphs that a firm in possession of no signal and seeing no investment must at all times have beliefs no higher than π_0, and hence must never find immediate investment profitable. $\square$

O.2 Auxiliary results about firm continuation values

In this appendix we provide several auxiliary results related to continuation values appearing in the HJB equation.

Lemma O.2. *For any μ,*

$$h(\mu)(\mu_+ R - 1) - c = K(\mu - \pi_A),$$

where $\mu_+ = q^H \mu / h(\mu)$ and

$$K \equiv q^H(R-1) + (1-q^L) > 0.$$

Proof. Some algebra shows that

$$h(\mu)(\mu_+ R - 1) - c = (q^H(R-1) + (1-q^L))\left(\mu - \frac{(1-q^L)+c}{q^H(R-1)+(1-q^L)}\right).$$

Recall π_A has the explicit representation $\pi_A = ((1-q^L)+c)/(q^H(R-1)+(1-q^L))$, establishing the identity. $\square$

Lemma O.3. $\overline{V} \leq K(\pi_+ - \pi_-)$.

Proof. If signals are complements, then $\overline{V} = \frac{\overline{\lambda}}{\overline{\lambda}+r} K(\pi_+ - \pi_A)$, in which case $\overline{V} < K(\pi_+ - \pi_A)$. If signals are substitutes, then $\overline{V} = \pi_+ R - 1$, and so by Assumption 3, $\overline{V} \leq K(\pi_+ - \pi_A)$. $\square$

O.3 Belief updating identities

In this appendix we derive several useful identities involving posterior beliefs about the state in the event no investment by the other firm has been observed. These identities will be used in proofs elsewhere in the online appendix.

Fix a firm i and any regular strategy for firm $-i$, and suppose that firm $-i$ does not randomize over its prospecting intensity and invests immediately upon receiving a High signal at all times. Let $\Omega^{-i}(t)$ be the (ex ante) probability that $-i$ has received no signal by time t. Then

$$\Omega^{-i}(t) = \exp\left(-\int_0^t \lambda^{-i}(s)\, ds\right).$$

Lemma O.4. *Suppose some firm $-i$ uses a regular strategy involving non-random prospecting and immediate investment upon receipt of a High signal. Then for almost every time t,*

$$\frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)} = -\lambda^{-i}(t)\frac{\mu^i(t) - \pi_-}{\pi_+ - \pi_-}.$$

Proof. Differentiating the definition of $\Omega^{-i}(t)$ yields

$$\dot{\Omega}^{-i}(t) = -\lambda^{-i}(t)\Omega^{-i}(t).$$

Meanwhile, by Bayes' rule

$$\mu^i(t) = \frac{(\Omega^{-i}(t) + (1-\Omega^{-i}(t))(1-q^H))\pi_0}{(\Omega^{-i}(t) + (1-\Omega^{-i}(t))(1-q^H))\pi_0 + (\Omega^{-i}(t) + (1-\Omega^{-i}(t))q^L)(1-\pi_0)}.$$

Using the identities $\pi_- = (1-q^H)\pi_0/l(\pi_0)$ in the numerator and $l(\pi_0) = (1-q^H)\pi_+ + q^L(1-\pi_0)$ in the denominator, this expression may be rewritten

$$\mu^i(t) = \frac{\Omega^{-i}(t)\pi_0 + (1-\Omega^{-i}(t))l(\pi_0)\pi_-}{\Omega^{-i}(t) + (1-\Omega^{-i}(t))l(\pi_0)}.$$

Solving this identity for $\Omega^{-i}(t)$ yields

$$\Omega^{-i}(t) = \frac{l(\pi_0)}{h(\pi_0)}\frac{\mu^i(t) - \pi_-}{\pi_+ - \mu^i(t)}.$$

Differentiating this expression and eliminating $\dot{\Omega}^{-i}(t)$ using the identity derived above yields the desired relationship. $\square$

Lemma O.5. *For every* $\mu \in [\pi_-, \pi_+]$,

$$h(\pi_+)\frac{\mu - \pi_-}{\pi_+ - \pi_-} = h(\mu)\frac{\mu_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}}.$$

Proof. Note that both the lhs and rhs of the identity in the lemma statement are affine functions of μ. (The lhs is immediate, while the numerator of the rhs may be rewritten $q^H\mu - \pi_{+-}h(\mu)$, which is affine in μ given that $h(\mu)$ is.) It is therefore enough to show that they coincide at two distinct values of μ. Note that when $\mu = \pi_-$, both sides vanish, while when $\mu = \pi_+$, both sides reduce to $h(\pi_+)$, as desired. □

Lemma O.6.

$$h(\pi_+) = \frac{\pi_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}}.$$

Proof. Using the Bayes' rule identities $\pi_{++} = q^H\pi_+/h(\pi_+)$ and $\pi_{+-} = (1 - q^H)\pi_+/l(\pi_+)$, some algebra yields

$$\frac{\pi_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}} = 1 - \frac{\pi_{++} - \pi_+}{\pi_{++} - \pi_{+-}} = 1 - \frac{\frac{q^H\pi_+}{h(\pi_+)} - \pi_+}{\frac{q^H\pi_+}{h(\pi_+)} - \frac{(1-q^H)\pi_+}{l(\pi_+)}} = 1 - l(\pi_+) = h(\pi_+).$$

□

In the following lemma, let S^i be the random variable representing the latent value of i's signal (whether or not i has yet acquired it).

Lemma O.7. *Suppose some firm* $-i$ *uses a regular strategy involving immediate investment upon receipt of a High signal. Then for any time* t,

$$\Pr(S^{-i} = H \mid S^i = H, \tau^{-i}(\emptyset) \geq t) = \frac{\mu^i_+(t) - \pi_{+-}}{\pi_{++} - \pi_{+-}}.$$

Proof. If firm $-i$ invests immediately upon receipt of a signal, and never invests following receipt of a low signal, then Bayes' rule implies that

$$\Pr(S^{-i} = H \mid S^i = H, \tau^{-i}(\emptyset) \geq t) = \frac{\Omega^{-i}(t)h(\pi_+)}{\Omega^{-i}(t)h(\pi_+) + l(\pi_+)}.$$

Meanwhile, another application of Bayes' rule yields

$$\mu^i_+(t) = \frac{(\Omega^{-i}(t) + (1 - \Omega^{-i}(t))(1 - q^H))\pi_+}{(\Omega^{-i}(t) + (1 - \Omega^{-i}(t))(1 - q^H))\pi_+ + (\Omega^{-i}(t) + (1 - \Omega^{-i}(t))q^L)(1 - \pi_+)}.$$

Using the identities $\pi_{+-} = (1 - q^H)\pi_+/l(\pi_+)$ in the numerator and $l(\pi_+) = (1 - q^H)\pi_+ + q^L(1 - \pi_+)$ in the denominator, this expression may be rewritten

$$\mu_+^i(t) = \frac{\Omega^{-i}(t)\pi_+ + (1 - \Omega^{-i}(t))l(\pi_+)\pi_{+-}}{\Omega^{-i}(t) + (1 - \Omega^{-i}(t))l(\pi_+)}.$$

This representation allows us to write

$$\frac{\mu_+^i(t) - \pi_{+-}}{\pi_{++} - \pi_{+-}} = \frac{\pi_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}} \frac{\Omega^{-i}(t)}{\Omega^{-i}(t) + (1 - \Omega^{-i}(t))l(\pi_+)}.$$

Using Lemma O.6, this identity may be equivalently written

$$\frac{\mu_+^i(t) - \pi_{+-}}{\pi_{++} - \pi_{+-}} = \frac{\Omega^{-i}(t)h(\pi_+)}{\Omega^{-i}(t)h(\pi_+) + l(\pi_+)} = \Pr(S^{-i} = H \mid S^i = H, \tau^{-i}(\emptyset) \geq t).$$

□

Lemma O.8. *Suppose some firm $-i$ uses a regular strategy involving non-random prospecting and immediate investment upon receipt of a High signal. Then for almost every time t,*

$$\Pr(s_t^{-i} = \emptyset \mid \tau^{-i}(\emptyset) \geq t)\lambda^{-i}(t) = -\frac{1}{h(\pi_0)} \frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)}.$$

Proof. By Bayes' rule,

$$\Pr(s_t^{-i} = \emptyset \mid \tau^{-i}(\emptyset) \geq t) = \frac{\Omega^{-i}(t)}{\Omega^{-i}(t) + (1 - \Omega^{-i}(t))l(\pi_0)}.$$

In the proof of Lemma O.4, we established the identity

$$\Omega^{-i}(t) = \frac{l(\pi_0)}{h(\pi_0)} \frac{\mu^i(t) - \pi_-}{\pi_+ - \mu^i(t)}.$$

Substituting into the previous expression to eliminate $\Omega^{-i}(t)$ in favor of $\mu^i(t)$ yields

$$\Pr(s_t^{-i} = \emptyset \mid \tau^{-i}(\emptyset) \geq t) = \frac{1}{h(\pi_0)} \frac{\mu^i(t) - \pi_-}{\pi_+ - \pi_-}.$$

Multiplying through by $\lambda^{-i}(t)$ and using the identity derived in Lemma O.4 yields the expression in the lemma statement. □

O.4 Proof of Proposition 3

Propositions 1 and 2 establish that the symmetric and leader-follower strategy profiles are sufficient for equilibrium. We now prove that that these profiles are also necessary for equilibrium.

We begin by establishing that any best response to a regular strategy features a threshold rule for investment.

Lemma O.9. *Suppose firm $-i$ uses a regular strategy. Let $T_i^0 \equiv \inf\{t \ : \ \mu_+^i(t) \leq 1/R\}$ and $\underline{T}_i \equiv \inf\{t \ : \ \mu_+^i(t) < 1/R\}$. Then:*

- *If $\underline{T}_i < \infty$, there exists a cutoff time $T_i^* \leq T_i^0$ such that every best reply for firm i after obtaining a High signal at time t entails investing immediately if $t < T_i^*$ and waiting forever if $t > T_i^*$,*
- *If $\underline{T}_i = \infty$,every best reply for firm i after obtaining a High signal at time t entails investing immediately if $t < T_i^0$. Upon receiving a signal at any $t \geq T_i^0$, any continuation strategy by firm i is a best reply.*

Proof. We show first that, for any time t such that $\mu_+^i(t) > 1/R$, any best reply for i either invests immediately or waits until $-i$ invests. Suppose by way of contradiction that firm i had a best reply such that upon receiving a High signal at time t, i invests at the random time $\widetilde{\tau}^i \in [t, \infty) \cup \{\infty\}$ conditional on no investment by firm $-i$, with $\Pr(\widetilde{\tau}^i \in (t, \infty)) > 0$. Then there must exist another best reply such that firm i waits until some deterministic time $t' \in (t, \infty)$ and then invests w.p. 1 at time t' conditional on no investment by firm $-i$. In particular, it must be that $\mu_+^i(t')R - 1 \geq 0$.

Let $\tau^{-i}(\emptyset)$ be the (possibly random) time at which firm $-i$ invests, absent investment by firm i. Whenever $\tau^{-i}(\emptyset) \geq t'$, i's ex post continuation payoff as of time t is $e^{-rt'}(\mu_+^i(t')R-1) \leq \mu_+^i(t')R - 1$. In particular, if $\Pr(\tau^{-i}(\emptyset) \geq t' \mid \tau^{-i}(\emptyset) \geq t, S^i = H) = 1$, then $\mu_+^i(t') = \mu_+^i(t)$ and the previous inequality is strict. And whenever $\tau^{-i}(\emptyset) \in [t, t']$, i's continuation payoff is $e^{-r\tau^{-i}}(\pi_{++}R - 1) < \pi_{++}R - 1$. Then i's continuation payoff from this best reply is strictly less than

$$\begin{aligned} U' = \ & \Pr(\tau^{-i}(\emptyset) < t' \mid \tau^{-i}(\emptyset) \geq t, S^i = H)(\pi_{++}R - 1) \\ & + \Pr(\tau^{-i}(\emptyset) \geq t' \mid \tau^{-i}(\emptyset) \geq t, S^i = H)(\mu_+^i(t')R - 1). \end{aligned}$$

As

$$\mu_+^i(t) = \Pr(\tau^{-i}(\emptyset) < t' \mid \tau^{-i}(\emptyset) \geq t, S^i = H)\pi_{++} + \Pr(\tau^{-i}(\emptyset) \geq t' \mid \tau^{-i}(\emptyset) \geq t, S^i = H)\mu_+^i(t'),$$

$U' = \mu^i_+(t)R - 1$, which is exactly i's payoff from investing immediately at time t. Thus waiting until t' and then investing cannot be cannot be a best reply, yielding the desired contradiction.

Suppose first that $\underline{T}_i < \infty$. Consider times t such that $\mu^i_+(t) \leq 1/R$. For any t such that $\mu^i_+(t) < 1/R$, trivially the unique best response is for i to wait forever, since investing at any time after t yields a strictly negative payoff. It is also true that whenever $\mu^i_+(t) = 1/R$, waiting forever is i's unique best response. For investing at any time before $-i$ invests yields a non-positive continuation payoff, whereas given $\underline{T}_i < \infty$ there is a positive probability that $-i$ invests in the future, so that waiting for $-i$ to invest yields a strictly positive payoff.

Next fix any time t for which investing immediately is a best response for i. In this case $\mu^i_+(t) > 1/R$. We claim that for every $t' < t$, investing immediately is i's unique best response. Suppose by way of contradiction that for some $t' < t$, there existed a best response which does not invest immediately. Then by the discussion above, waiting until $-i$ invests must be a best response. But by assumption upon reaching time t with no investment by $-i$, investing immediately is a best response. Hence waiting until time t and then investing must also be a best response. This is a contradiction of earlier results, as desired.

Hence when $\underline{T}_i < \infty$, the set of best responses by i must have a simple structure - there exists a cutoff time $T^*_i \leq T^0_i$ such that any best response by i invests immediately for $t < T^*_i$ and waits for any $t > T^*_i$.

Now consider the case $\underline{T}_i = \infty$. Suppose first that $T^0_i < \infty$. For times $t \geq T^0_i$, the firm's beliefs upon obtaining a High signal are fixed at $1/R$, and so any investment strategy is optimal. In particular, investing immediately is optimal. So suppose by way of contradiction that for some $t < T^0_i$, firm i optimally waits forever to invest. Then another best reply is to wait until time T^0_i and then invest, a contradiction of earlier results. So it must be that for every $t < T^0_i$, immediate investment is optimal. Finally, suppose $T^0_i = \infty$. By a variant of the arguments above, it can be shown that waiting to invest delays a profitable investment when $-i$ does eventually invest, without avoiding any losses when $-i$ does not eventually invest, given that $\mu^i_+(\infty) \geq 1/R$. So waiting forever must yield strictly lower profits than investing immediately at all times. $\square$

This lemma does not entirely rule out existence of best replies which do not take a threshold form. The one case which in other best replies exist is if, for some firm i, $\mu^i_+(t)$ eventually drops to $1/R$ and then stays fixed there forever. In this case any investment policy by firm i is optimal once beliefs have dropped to $1/R$. However, no best reply by firm i ever leaves it in possession of a High signal in such a history. This is because either the firm obtained a High signal earlier and invested; or else the firm has not obtained a signal and $\mu^i_+(t) = 1/R$, in which case a signal has no value and the firm would never optimally acquire one at that point. Therefore any other best replies differ only off-path, regardless of the strategy chosen by firm $-i$.

Our proof will proceed by restricting attention to equilibria in threshold investment strategies. This restriction will not rule out any equilibrium paths, and therefore we will be able to determine whether there exist any equilibria in which $\mu^i_+(t)$ is eventually fixed at $1/R$ for some i. It will turn out there are not, and thus we do not exclude any equilibria by restricting attention to threshold investment strategies. We will further restrict attention to equilibria in pure prospecting strategies. At the end of the proof we verify that no equilibria with mixed prospecting strategies can exist.

Given any belief process μ^i, define

$$t_i^A \equiv \inf\{t \ : \ \mu^i(t) \leq \pi_A\}$$

to be the time at which firm i's beliefs reach the autarky threshold. We first establish an important technical result about the dynamics of the value of effort prior to time t_i^A. This result will be critical to establishing that firms follow a threshold rule in effort in any equilibrium.

Lemma O.10. *Fix any firm i and any regular threshold investment strategy by firm $-i$. Let V^i be firm i's value function given firm $-i$'s strategy, and define $f_i(t) \equiv V^i(t) - K(\mu^i(t) - \pi_A)$. Then for almost every $t \in [0, \min\{T_i^*, t_i^A\}]$, either $f_i(t) < 0$ or $f_i'(t) > 0$.*

Proof. Fix a firm i. Suppose first that $T^*_{-i} \leq t < t_i^A$. Then at time t firm i is in autarky with beliefs $\mu^i(t) > \pi_A$, meaning its continuation value is $V^i(t) = \frac{\overline{\lambda}}{\overline{\lambda}+r} K(\mu^i(t) - \pi_A) < K(\mu^i(t) - \pi_A)$. Thus $f_i(t) < 0$ for all such times. So it is sufficient to establish the result for $t < \hat{t}_i \equiv \min\{T_i^*, t_i^A, T_{-i}^*\}$.

First note that as V^i and μ^i are both absolutely continuous, f_i is absolutely continuous as well and f_i' is defined a.e. Let

$$T^* = \{t \in [0, \hat{t}_i) \ : \ f_i(t) \geq 0\}.$$

For almost every $t \in T^*$, the HJB equation

$$rV^i(t) = -\frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)}(\overline{V} - V^i(t)) + \dot{V}^i(t)$$

must hold. This may be rewritten in terms of f and f' as

$$f_i'(t) = rf_i(t) + rK(\mu^i(t) - \pi_A) + \frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)}(\overline{V} - K(\pi_+ - \pi_A) - f_i(t)).$$

As $f_i(t) \geq 0$ on T^*, the inequality

$$f_i'(t) \geq rK(\mu^i(t) - \pi_A) + \frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)}(\overline{V} - K(\pi_+ - \pi_A))$$

must hold almost everywhere on T^*. Now use the identity

$$\frac{\dot{\mu}^i(t)}{\pi_+ - \mu^i(t)} = -\lambda^{-i}(t)\frac{\mu^i(t) - \pi_-}{\pi_+ - \pi_-},$$

which holds for every $t < T^*_{-i}$, to rewrite this inequality as

$$f_i'(t) \geq \frac{\mu^i(t) - \pi_-}{\pi_+ - \pi_-}\left(-\lambda^{-i}(t)(\overline{V} - K(\pi_+ - \pi_A)) + rK(\mu^i(t) - \pi_A)\frac{\pi_+ - \pi_-}{\mu^i(t) - \pi_-}\right).$$

Regardless of firm $-i$'s strategy, $\mu^i(t) > \pi_-$ for all time. So the lemma is proven if we can show that the final term on the rhs is strictly positive for $t < \hat{t}_i$. By Lemma O.3, $\overline{V} \leq K(\pi_+ - \pi_A)$. So the first term on the rhs is non-negative. Meanwhile t_i^A implies that the second term on the rhs is strictly positive. So $f_i'(t) > 0$. □

We proceed by splitting the analysis into two cases: either $T_i^* < \infty$ for some firm i, or else $T_1^* = T_2^* = \infty$. We will show that in the first case, the only permissible equilibrium behavior is the leader-follower strategy profile, while in the second case, the only permissible behavior is the symmetric equilibrium profile. Consider first the $T_i^* < \infty$ case. The following lemma establishes that the remaining firm $-i$ must employ the leader strategy in any equilibrium.

Lemma O.11. *Fix any perfect Bayesian equilibrium in threshold investment strategies such that $T_i^* < \infty$ for some firm i. Then firm $-i$'s strategy must be the threshold strategy $\overline{T}_{-i} = T^*_{-i} = \infty$.*

We establish this result by first proving a series of auxiliary lemmas which restrict the permissible scope of equilibrium behavior and beliefs in response to a firm using a threshold investment rule with $T_i^* < \infty$.

Lemma O.12. *Fix any perfect Bayesian equilibrium in threshold investment strategies such that $T_i^* < \infty$ for some firm i. Then $T^*_{-i} = \infty$ and $\mu^{-i}(T_i^*) \geq \pi_A$.*

Proof. Without loss take $i = 1$. Suppose by way of contradiction that $\mu^2(T_1^*) < \pi_A$. In this case clearly $T_1^* > 0$ given that $\mu^2(0) = \pi_0 > \pi_A$. Further, as the belief process is continuous under a threshold investment strategy it must be that $t_2^A < T_1^*$. Then as $V^2 \geq 0$, for $t \in (t_2^A, T_1^*]$ it must be that $V^2(t) > K(\mu^2(t) - \pi_A)$, so that by the HJB equation 2's essentially unique optimal prospecting policy is $\lambda^2(t) = 0$ for $t \in (t_2^A, T_1^*]$. And after T_1^* firm 2 is in autarky with beliefs below the autarky threshold, so it must also be that $\lambda^2(t) = 0$ for almost every $t \geq T_1^*$.

Then on the equilibrium path firm 2 never invests first after t_2^A, meaning firm 1 is in autarky with constant beliefs $\mu^1(t) = \mu^1(t_2^A)$ for all $t > t_2^A$. But then $\mu_+^1(t_2^A) = 1/R$, otherwise it could not be optimal for firm 1 to invest immediately prior to T_1^* and wait forever after T_1^*. In this case $\mu^1(t_2^A) < \pi_A$, so $\lambda^1(t) = 0$ for all $t \geq t_2^A$. But then on the equilibrium path firm 1 does not invest first on $[t_2^A, T_1^*]$, implying $\mu^2(t_2^A) = \mu^2(T_1^*)$. This contradicts our assumption on $\mu^2(T_1^*)$ and the definition of t_2^A. So it must be that $\mu^2(T_1^*) \geq \pi_A$.

As firm 1 does not invest first on the equilibrium path after T_1^*, firm 2 is in autarky with constant beliefs $\mu^2(t) \geq \pi_A$ for all $t \geq T_1^*$. Hence $\mu_+^2(t) > 1/R$ and so immediate investing is strictly superior to waiting forever for every $t \geq T_1^*$. In other words, $T_2^* = \infty$. $\square$

Lemma O.13. *Fix any perfect Bayesian equilibrium in threshold investment strategies in which $T_i^* < \infty$ for some firm i. Then $\mu^{-i}(T_i^*) > \pi_A$.*

Proof. Without loss take $i = 1$. By Lemma O.12 we already know that $\mu^2(T_1^*) \geq \pi_A$ and $T_2^* = \infty$. Assume by way of contradiction that $\mu^2(T_1^*) = \pi_A$. Clearly $T_1^* > 0$ in this case given $\mu^2(0) = \pi_0 > \pi_A$, and further $t_2^A \leq T_1^*$ given the definition of t_2^A.

Next observe that in equilibrium, firm 1 never invests first after t_2^A. This is automatically true for $t \geq T_1^*$, so the remaining thing to show is that $\lambda^1(t) = 0$ on $[t_2^A, T_1^*)$ in the case that this interval is non-empty. But $\mu^2(t)$ is constant on this interval, so given that firm 1 invests immediately upon obtaining a High signal its prospecting rate must be zero. Thus $V^2(t_2^A) = 0$, since at time t_2^A firm 2 is in autarky with beliefs π_A. Also $V^2(t) > 0$ for all $t < t_2^A$. For at any such time $\mu^2(t) > \pi_A$, so firm 2's autarky payoff as of time t is strictly positive, and this payoff is a lower bound on $V^2(t)$.

Define $f_2(t) \equiv V^2(t) - K(\mu^2(t) - \pi_A)$. By Lemma O.10, for almost every $t \in [0, t_2^A]$ either $f_2(t) < 0$ or $f_2'(t) > 0$. Note also that $V^2(t_2^A) = 0$ and $\mu^2(t_2^A) = \pi_A$ implies $f_2(t_2^A) = 0$. We next establish that $\lambda^2(t) = \overline{\lambda}$ a.e. on $[0, t_2^A]$.

Suppose first that $f_2(t') > 0$ for some $t' \in [0, t_2^A)$. Then as $f_2(t_2^A) = 0$ and f_2 is absolutely continuous, there must exist a positive-measure subset of $[t', t_2^A]$ on which $f_2(t) > 0$ and $f_2'(t) < 0$, a contradiction. So certainly $f_2(t) \leq 0$ on $[0, t_2^A]$. If $f_2(t) = 0$ on a positive-measure subset of $[0, t_2^A]$, then a.e. on this set $f_2'(t) > 0$. But whenever $f_2(t) = 0$ and

$f_2'(t) > 0$, the definition of $f_2'(t)$ implies that $f_2(t+\varepsilon) > f_2(t) = 0$ for sufficiently small $\varepsilon > 0$, a contradiction. So $f_2(t) < 0$ for almost every $t \in [0, t_2^A]$. Hence from the HJB equation $\lambda^2(t) = \overline{\lambda}$ for almost every $t \in [0, t_2^A]$.

This means in particular that $\mu^1(t) \leq \mu^2(t)$ for $t \in [0, t_2^A]$ and therefore $t_1^A \leq t_2^A$, no matter the prospecting policy firm 1 follows. If $t_1^A < t_2^A$, then the fact that firm 2 prospects with positive intensity and invests immediately on $[t_1^A, t_2^A]$ means $V^1(t_1^A) > 0$. Then as $\mu^1(t) \leq \pi_A$ for $t \geq t_1^A$, the HJB equation requires that $\lambda^1(t) = 0$ for almost every $t \in [t_1^A, t_2^A]$. But then μ^2 is constant on this interval, a contradiction of the fact that t_2^A is the first time μ^2 hits π_A. So $t_1^A = t_2^A = t^A$, which can only hold if $\lambda^1(t) = \overline{\lambda}$ for almost every $t \in [0, t^A]$.

If $V^1(t^A) > 0$, then given continuity of V^1 and μ^1, for sufficiently large $t < t^A$ it would be the case that $V^1(t) > K(\mu^1(t) - \pi_A)$. But then $\lambda^1(t) = 0$ by the HJB equation, a contradiction. So $V^1(t^A) = 0$. But as $T_2^* = \infty$ by Lemma O.12, this can only be true if $\lambda^2(t) = 0$ for a.e. $t > t^A$. As $\mu^1(t_1^A) = \pi_A$ by definition of t_1^A, it must be that $\mu^1(t) = \pi_A$ for all $t > t^A$, so $\mu_+^1(t) > 1/R$ on this time range. As firm 2 never invests along the equilibrium path after t^A, this means that investing immediately upon receiving a High signal must yield a strictly higher continuation payoff for firm 1 than waiting forever, contradicting $T_1^* < \infty$. This is the desired contradiction ruling out $\mu^2(T_1^*) = \pi_A$. $\square$

Proof of Lemma O.11. Without loss suppose $i = 1$. By Lemma O.12 $T_2^* = \infty$, and by Lemma O.13 $\mu^2(T_1^*) > \pi^A$. The latter inequality implies that for $t > T_1^*$ firm 2 is in autarky with constant beliefs $\mu^2(t) = \mu^2(T_1^*) > \pi_A$, meaning 2's unique optimal prospecting policy subsequent to T_1^* is $\lambda^2(t) = \overline{\lambda}$. It remains only to pin down firm 2's optimal prospecting behavior prior to T_1^*.

We proceed by establishing that $V^\dagger(t) = K(\mu^2(t) - \pi_A)$ is a strict supersolution to firm 2's HJB equation on $[0, T_1^*]$. Recall that the HJB equation for firm 2 in this regime is

$$rV^2(t) = \overline{\lambda}\left(K(\mu^2(t) - \pi_A) - V^2(t)\right)_+ - \frac{\dot{\mu}^2(t)}{\pi_+ - \mu^2(t)}\left(\overline{V} - V^2(t)\right) + \dot{V}^2(t).$$

So define the functional

$$F(w,t) \equiv rw(t) - \overline{\lambda}\left(K(\mu^2(t) - \pi_A) - w(t)\right)_+ + \frac{\dot{\mu}^2(t)}{\pi_+ - \mu^2(t)}\left(\overline{V} - w(t)\right) - \dot{w}(t).$$

The claim that $V^\dagger$ is a strict supersolution is equivalent to $F(V^\dagger, t) > 0$ for $t < T_1^*$. Evaluating the functional at $V^\dagger$ yields

$$F(V^\dagger, t) = rV^\dagger(t) + \frac{\dot{\mu}^2(t)}{\pi_+ - \mu^2(t)}(\overline{V} - K(\pi_+ - \pi_A)).$$

Note that $\overline{V} \leq K(\pi_+ - \pi_A)$ by Lemma O.3, so the second term on the rhs is non-negative. Meanwhile $\mu^2(t) > \pi_A$ for $t \leq T_1^*$, so $F(V^\dagger, t) > 0$ as claimed.

Now note that as firm 2 is in autarky at time T_1^*, its value function at this point is $V^2(T_1^*) = \frac{\overline{\lambda}}{\overline{\lambda}+r}K(\mu^2(T_1^*) - \pi_A) < V^\dagger(T_1^*)$. This boundary condition combined with the fact that $V^\dagger$ is a strict supersolution implies $V^\dagger(t) > V^2(t)$ for all $t \in [0, T_1^*]$. The HJB equation then implies that $\lambda^2(t) = \overline{\lambda}$ is the unique best reply for firm 2. In other words, in equilibrium firm 2 must employ a threshold prospecting strategy with $\overline{T}_2 = \infty$. □

Lemma O.11 establishes that in any equilibrium in threshold investment strategies in which some $T_i^* < \infty$, then the other firm must follow the leader's strategy. Meanwhile Lemma B.3 establishes that the follower's strategy is a unique best reply to the leader's strategy. So there exists a unique equilibrium in threshold investment strategies with some $T_i^* < \infty$, namely the leader-follower equilibrium.

The following lemma treats the remaining case, in which $T_1^* = T_2^* = \infty$. It establishes that the symmetric equilibrium strategies are the only ones consistent with equilibrium in this case.

Lemma O.14. *Fix any perfect Bayesian equilibrium in threshold investment strategies in which $T_1^* = T_2^* = \infty$. Then $\lambda^1(t) = \lambda^2(t) = \overline{\lambda}$ for every $t < T^A$, while $\lambda^1(t) = \lambda^2(t) = 0$ for every $t > T^A$.*

Proof. Let $f_i(t) \equiv V^i(t) - K(\mu^i(t) - \pi_A)$ be the term in firm i's HJB equation whose sign determines i's optimal prospecting rate. We show first that each $t_i^A < \infty$. To establish this, maintain for the time being the opposite assumption, that some $t_i^A = \infty$, say $i = 1$.

By Lemma O.10, for almost every t either $f_1(t) < 0$ or $f_1'(t) > 0$. If $f_1(t) < 0$ a.e., then the HJB equation would imply that $\lambda^1(t) = \overline{\lambda}$ a.e. But in this case eventually $\mu_+^2(t) < 1/R$, meaning $t_2^* < \infty$, a contradiction of $t_1^* = \infty$. So on some positive-measure set of times $T^\dagger$, $f_1(t) \geq 0$ and $f_1'(t) > 0$. But whenever $f_1(t) \geq 0$ and $f_1'(t) > 0$, the definition of $f_1'(t)$ implies that $f_1(t + \varepsilon) > 0$ for sufficiently small $\varepsilon > 0$. Therefore $t^0 \equiv \inf\{t \ : \ f_1(t) > 0\} < \infty$.

Suppose $f_1(t') \leq 0$ for some $t' > t^0$. Then by definition of t^0 there exists a $t'' < t'$ such that $f_1(t'') > 0$, and so absolute continuity of f_1 implies that $f_1(t) > 0$ and $f_1'(t) < 0$ on some positive-measure subset of $[t'', t']$. This contradicts our earlier finding, so $f_1(t) > 0$ for all $t > t^0$. Hence the HJB equation implies that $\lambda^1(t) = 0$ for a.e. $t > t^0$.

Further, by definition of t^0 it must be that $f_1(t) \leq 0$ for all $t < t^0$. If $f_1(t) = 0$ on some positive-measure subset of $[0, t^0)$, then $f_1'(t) > 0$ a.e. on this set and so there would exist a $t' < t^0$ such that $f_1(t') > 0$ a contradiction. Hence $f_1(t) < 0$ for almost every $t \in [0, t^0]$, implying by the HJB equation that $\lambda^1(t) = \overline{\lambda}$ almost everywhere on $[0, t^0]$.

The fact that firm 1 does not prospect subsequent to t^0 means that firm 2 is in autarky after t^0. If $\mu^2(t^0) < \pi_A$, then its optimal prospecting rate is 0. But the assumption of $t_1^A = \infty$ means $\mu^1(t^0) > \pi_A$, so firm 1 would also also be autarky after t^0 with beliefs above the autarky threshold, contradicting the optimality of $\lambda^1(t) = 0$. On the other hand, if $\mu^2(t^0) > \pi_A$, then firm 2's optimal prospecting rate after t^0 is $\overline{\lambda}$ forever. But in this case eventually $\mu_+^1(t) < 1/R$, meaning $t_1^* < \infty$, a contradiction of our assumption. So it must be that $\mu^2(t^0) = \pi_A$. This implies in particular that $V^2(t^0) = 0$ and $f_2(t^0) = 0$ given that firm 2 is in autarky after t^0.

Meanwhile $\mu^2(t) > \pi_A$ for all $t < t^0$ given that firm 1 prospects at a strictly positive rate and invests immediately until t^0. So $t^0 = t_2^A$, and Lemma O.10 tells us that for almost every $t \in [0, t^0]$, either $f_2(t) < 0$ or $f_2'(t) > 0$. If ever $f_2(t') > 0$ for some $t' \in [0, t^0)$, then $f_2(t^0) = 0$ and absolute continuity of f_2 would imply $f_2(t) > 0$ and $f_2'(t) < 0$ on a positive-measure subset of $[t', t^0]$, a contradiction. So $f_2(t) \leq 0$ on $[0, t^0]$, and further $f_2(t) < 0$ for almost every $t \in [0, t^0]$. Thus by the HJB equation $\lambda^2(t) = \overline{\lambda}$ a.e. on $[0, t^0]$, implying $\mu^1(t^0) = \pi_A$ given that $\mu^2(t^0) = \pi_A$ and both firms employ the same prospecting and investing strategy prior to t^0. This yields the desired contradiction of our hypothesis that $t_1^A = \infty$.

So it must be that each $t_i^A < \infty$. Wlog we assume that $t_1^A \leq t_2^A$ going forward. We next prove that $V^1(t_1^A) = 0$. For the time being, suppose instead that $V^1(t_1^A) > 0$.

Given the hypothesis on $V^1(t_1^A)$, the definition of t_1^A, and the continuity of V^1 and μ^1, for sufficiently large $t < t_1^A$ it must be that $f_1(t) > 0$. Hence $\lambda^1(t) = 0$ from the HJB equation, meaning $\mu^2(t)$ is constant and therefore $t_2^A > t_1^A$. If there existed a $t' \in (t_1^A, t_2^A)$ such that $\mu^1(t) < \pi_A$, then $\lambda^1(t) = 0$ a.e. on $[t', t_2^A]$, meaning $\mu^2(t)$ would be constant on that interval, a contradiction of the definition of t_2^A. So $\mu^1(t) = \pi_A$ on $[t_1^A, t_2^A]$, reducing the HJB equation for V^1 to $rV^1(t) = \dot{V}^1(t)$, with solution $V^1(t) = e^{r(t - t_1^A)} V^1(t_1^A)$. So $V^1(t) > 0$ on $[t_1^A, t_2^A]$, meaning that $\lambda^1(t) = 0$ a.e. on the interval, yielding a constant μ^2 and another contradiction. So it must be that $V^1(t_1^A) = 0$ and hence also $f_1(t_1^A) = 0$.

Given $f_1(t_1^A) = 0$, a nearly identical argument to that applied to f_2 prior to t^0 implies that $\lambda^1(t) = \overline{\lambda}$ a.e. on $[0, t_1^A]$. Then surely $t_2^A \leq t_1^A$ no matter what prospecting policy firm 2 chooses, meaning $t_1^A = t_2^A$ and $\lambda^2(t) = \overline{\lambda}$ for $t \leq t_1^A = t_2^A$. Given these prospecting strategies, it must be that $t_1^A = t_2^A = T^A$.

Finally, suppose that $\lambda^i(t) > 0$ for some i and some positive-measure subset of $[T^A, \infty)$.

Then given $t_i^* = \infty$, firm $-i$'s value at T^A would be strictly positive. But then for sufficiently large $t < T^A$ we would have $f_{-i}(t) > 0$, contradicting $\lambda^{-i} = \overline{\lambda}$. So it must be that $\lambda^1(t) = \lambda^2(t) = 0$ a.e. on $[T^A, \infty)$. $\square$

We complete the proof by ruling out mixed prospecting rules in equilibrium. This is accomplished by the following lemma, which establishes that any equilibrium involving randomization over prospecting co-exists with a pure-strategy equilibrium involving interior prospecting. As none of the equilibria characterized above exhibit such behavior, it was without loss to ignore the possibility of mixed prospecting strategies.

Lemma O.15. *Fix any perfect Bayesian in threshold investment strategies. Then there exists a payoff-equivalent perfect Bayesian equilibrium in pure strategies, exhibiting interior prospecting for any firm and time at which the firm randomized over prospecting in the original equilibrium.*

Proof. Fix a perfect Bayesian equilibrium in threshold investment strategies. As threshold investment strategies are automatically pure strategies, we need only consider randomization over prospecting. Suppose that some firm $-i$ mixes over prospecting, with prospecting rule λ^{-i} conditioning on firm $-i$'s randomization device. Let T^*_{-i} be firm $-i$'s cutoff time for investment. After this time, firm $-i$'s prospecting rule does not affect firm i's payoffs or incentives at all; thus λ^{-i} may be replaced with any pure strategy maximizing $-i$'s payoffs subsequent to time T^*_{-i} without disturbing the equilibrium. So consider times $t < T^*_{-i}$.

Generalize the definition of Ω^{-i} from Appendix O.3 by letting

$$\Omega^{-i}(t) = \mathbb{E}\left[\exp\left(-\int_0^t \lambda^{-i}(s)\,ds\right)\right].$$

Define a new pure-strategy prospecting rule $\widetilde{\lambda}^{-i}$ by letting

$$\widetilde{\lambda}^{-i}(t) = -\frac{d}{dt}\log\Omega^{-i}(t)$$

for all times (with the prospecting rule arbitrary at any point of non-differentiability of Ω^{-i}.) The two strategies λ^i and $\widetilde{\lambda}^i$ induce the same sequence of posterior beliefs for firm i about the state conditional on observing no investment, by construction. Further, both prospecting rules induce the same distribution of investment times by $-i$. Thus firm i's incentives are unchanged by replacing λ^{-i} with $\widetilde{\lambda}^{-i}$.

It remains to check that $\widetilde{\lambda}^{-i}$ is both feasible and optimal for firm $-i$. Note that

$$\widetilde{\lambda}^{-i}(t) = \frac{1}{\Omega^{-i}(t)} \mathbb{E}\left[\lambda^{-i}(t) \exp\left(-\int_0^t \lambda^{-i}(s)\, ds\right)\right].$$

The second factor on the rhs is bounded above by $\overline{\lambda}\Omega^{-i}(t)$ and below by zero, hence $\widetilde{\lambda}^{-i}(t) \in [0, \overline{\lambda}]$, ensuring feasibility. As for optimality, suppose first that at time t, the action $\lambda^{-i}(t)$ is strictly optimal for firm $-i$. Then it must be non-random, in which case

$$\widetilde{\lambda}^{-i}(t) = \frac{1}{\Omega^{-i}(t)} \lambda^{-i}(t) \mathbb{E}\left[\exp\left(-\int_0^t \lambda^{-i}(s)\, ds\right)\right] = \lambda^{-i}(t).$$

So at any times for which randomization is not optimal for firm $-i$, the modified prospecting rule specifies the same prospecting intensity as the original rule. And at all other times, any prospecting intensity is optimal, thus in particular the intensity specified by $\widetilde{\lambda}^{-i}$ is optimal. So $\widetilde{\lambda}^{-i}$ is an optimal prospecting rule.

This argument shows that firm $-i$'s randomized prospecting rule may be replaced by a non-random one without disturbing payoffs, the optimality of $-i$'s strategy, or firm i's incentives. This procedure may be performed for both firms, yielding a pure strategy PBE.

Finally, for any time t at which $\lambda^{-i}(t)$ is not deterministic, it must be that $\Pr(\lambda^{-i}(t) > 0) > 0$ and $\Pr(\lambda^{-i}(t) < \overline{\lambda}) > 0$, meaning $\widetilde{\lambda}^{-i}(t) \in (0, \overline{\lambda})$. So randomization by some firm at some time in the original equilibrium yields an interior prospecting rate by that firm at the same time in the new equilibrium. □

O.5 Proofs of auxiliary lemmas related to Proposition 2

We begin with several technical lemmas characterizing quantities used in the proofs to follow.

Lemma O.16. *For every $\mu \in [\pi_-, \pi_0]$,*

$$\Delta(\mu) = \frac{\mu_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}} \frac{\overline{\lambda}}{\overline{\lambda} + r}(\pi_{++}R - 1) - (\mu_+ R - 1).$$

Proof. Suppose firm i receives a High signal at time t when its posterior beliefs are $\mu^i(t) = \mu$. The continuation value of investing immediately is exactly $\mu_+ R - 1$. Meanwhile, given that firm $-i$ employs the leader strategy, the value of waiting for firm $-i$ to invest is

$$\Pr(S^{-i} = H \mid S^i = H, \tau^{-i}(\emptyset) \geq t) \frac{\overline{\lambda}}{\overline{\lambda} + r}(\pi_{++}R - 1),$$

where the expression following the probability is the discounted value of eventual investment conditional on firm $-i$ eventually receiving a High signal. In Lemma O.7 we show that

$$\Pr(S^{-i} = H \mid S^i = H, \tau^{-i}(\emptyset) \geq t) = \frac{\mu_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}},$$

establishing the expression for Δ in the lemma statement. $\square$

Suppose firm i has current posterior beliefs $\mu \in [\pi_-, \pi_0]$ about the state, following a history in which it has no signal and has not seen firm $-i$ invest. Further suppose firm $-i$ employs the leader strategy. Under these conditions, let $\check{V}(\mu)$ be firm i's expected continuation value after receiving a signal, averaging over uncertainty in the realized signal. By definition of Δ,

$$\check{V}(\mu) = h(\mu)(\mu_+ R - 1 + \max\{\Delta(\mu), 0\}).$$

The term $h(\mu)$ reflects the probability that firm i's signal is High given its current posterior, and the remaining terms reflect the choice between investing immediately or waiting for the other firm to invest. (Lemma O.9 establishes that the firm's optimal continuation strategy must be one of these two choices.)

Lemma O.17. *For all* $\mu \in [\pi_-, \pi_0]$,

$$\check{V}(\mu) = \max\left\{h(\mu)(\mu_+ R - 1), \frac{\mu - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r} h(\pi_+)(\pi_{++} R - 1)\right\}.$$

Proof. When $\Delta(\mu) \leq 0$, the definition of $\check{V}$ implies that $\check{V}(\mu) = h(\mu)(\mu_+ R - 1)$. Meanwhile when $\Delta(\mu) > 0$, the definition implies that

$$\check{V}(\mu) = h(\mu)\frac{\mu_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}}\frac{\overline{\lambda}}{\overline{\lambda} + r}(\pi_{++} R - 1).$$

In Lemma O.5, we prove the identity

$$h(\pi_+)\frac{\mu - \pi_-}{\pi_+ - \pi_-} = h(\mu)\frac{\mu_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}},$$

establishing the expression for $\check{V}$ in the lemma statement. $\square$

Lemma O.18. *For all* $\mu \in [\pi_-, \pi_0]$,

$$\widetilde{\Delta}(\mu) = \frac{\mu - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} - \check{V}(\mu) + c.$$

Proof. By definition,

$$\widetilde{\Delta}(\mu) = -\left(\check{V}(\mu) - c - \Pr\left(s^{-i}_{(\mu^{\overline{\lambda}})^{-1}(\mu)} = \emptyset \mid \tau^{-i}(\emptyset) \geq (\mu^{\overline{\lambda}})^{-1}(\mu)\right)\frac{\overline{\lambda}}{\overline{\lambda}+r}h(\pi_0)\overline{V}\right).$$

The first term is the continuation value of obtaining a signal, the second term is the marginal cost of prospecting, and the final term is the firm's continuation value if no signal is obtained and it returns to its specified strategy of performing no prospecting until $-i$ invests. Combining Lemmas O.8 and O.4 allows $\Pr\left(s^{-i}_{(\mu^{\overline{\lambda}})^{-1}(\mu)} = \emptyset \mid \tau^{-i}(\emptyset) \geq (\mu^{\overline{\lambda}})^{-1}(\mu)\right)$ to be written explicitly, yielding the expression for $\widetilde{\Delta}(\mu)$ in the lemma statement. □

O.5.1 Proof of Lemma B.1

Differentiating Δ yields

$$\Delta'(\mu) = \left(\frac{1}{\pi_{++} - \pi_{+-}}\frac{\overline{\lambda}}{\overline{\lambda}+r}(\pi_{++}R - 1) - R\right)\frac{d\mu_+}{d\mu}.$$

By assumption $\pi_{+-} < 1/R < \pi_{++}$, so

$$\Delta'(\mu) < -\frac{r}{\overline{\lambda}+r}R\frac{d\mu_+}{d\mu} < 0.$$

Further, $\Delta(\pi_-) = -(\pi_{+-}R - 1) > 0$. Finally,

$$\Delta(\pi_0) = \frac{\pi_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}}\frac{\overline{\lambda}}{\overline{\lambda}+r}(\pi_{++}R - 1) - (\pi_+ R - 1),$$

and in Lemma O.6 we establish that

$$h(\pi_+) = \frac{\pi_+ - \pi_{+-}}{\pi_{++} - \pi_{+-}}.$$

So

$$\Delta(\pi_0) = \frac{\overline{\lambda}}{\overline{\lambda}+r}h(\pi_+)(\pi_{++}R - 1) - (\pi_+ R - 1).$$

When signals are complements

$$\frac{\overline{\lambda}}{\overline{\lambda}+r}h(\pi_+)(\pi_{++}R - 1) \geq \pi_+ R - 1 + \frac{\overline{\lambda}}{\overline{\lambda}+r}c > \pi_+ R - 1,$$

so $\Delta(\pi_0) > 0$.

O.5.2 Proof of Lemma B.2

Let

$$\hat{\Delta}(\mu) \equiv \frac{\mu - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r}(\overline{V} - h(\pi_+)(\pi_{++}R - 1)) + c.$$

Differentiate $\hat{\Delta}$ to obtain

$$\hat{\Delta}'(\mu) = \frac{1}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r}(\overline{V} - h(\pi_+)(\pi_{++}R - 1)).$$

By Lemma O.3, $\overline{V} \leq K(\pi_+ - \pi_A)$, i.e. $\overline{V} - h(\pi_+)(\pi_{++}R - 1) \leq -c$ and so $\hat{\Delta}'(\mu) < 0$ for all μ.

Now, $\widetilde{\Delta}(\mu) = \hat{\Delta}(\mu)$ for $\mu \leq \mu^*$, while $\widetilde{\Delta}(\mu) \leq \hat{\Delta}(\mu)$ for $\mu > \mu^*$. Clearly $\widetilde{\Delta}'(\mu) < 0$ for $\mu < \mu^*$. Meanwhile as $\widetilde{\Delta}$ is continuous at μ^* and an affine function of μ on $[\mu^*, \pi_0]$, to ensure $\widetilde{\Delta} \leq \hat{\Delta}$ it must be that $\widetilde{\Delta}'(\mu) = \widetilde{\Delta}'(\mu^*+) \leq \hat{\Delta}'(\mu^*) < 0$ for $\mu \in (\mu^*, \pi_0]$. Hence $\widetilde{\Delta}$ is a strictly decreasing function. Finally, note that $\widetilde{\Delta}(\pi_-) = c > 0$.

O.5.3 Proof of Lemma B.3

Assume firm $-i$ employs the strategy of the lemma statement. As firm $-i$'s strategy is regular, by Lemma O.9 firm i's optimal investment strategy involves a cutoff time T_i^* before which it invests immediately and after which it waits for firm $-i$ to invest. Further, given firm $-i$'s strategy, $\mu^i = \mu^{\overline{\lambda}}$. Thus by the definition of μ^*, the time $(\mu^{\overline{\lambda}})^{-1}(\mu^*)$ is exactly when the value to firm i of investing immediately falls below the value of waiting for firm $-i$ to invest first. So the firm's unique optimal investment strategy sets $T_i^* = (\mu^{\overline{\lambda}})^{-1}(\mu^*)$.

Next we derive firm i's optimal prospecting strategy. Appendix A establishes that firm i's continuation value function V^i satisfies

$$rV^i(t) = \overline{\lambda}\left(\check{V}(\mu^{\overline{\lambda}}(t)) - c - V^i(t)\right)_+ - \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}(\overline{V} - V^i(t)) + \dot{V}^i(t)$$

for all time. Define the functional

$$F(w,t) \equiv rw - \overline{\lambda}\left(\check{V}(\mu^{\overline{\lambda}}(t)) - c - w(t)\right)_+ + \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}\left(\overline{V} - w(t)\right) - \dot{w}(t).$$

We now show that $V^\dagger(t) \equiv \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}\overline{V}$ satisfies the HJB equation for $t \geq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$. In other words, $F(V^\dagger, t) = 0$ for such times. By definition of $\overline{\mu}$, $\check{V}(\mu^{\overline{\lambda}}(t)) - c - V^\dagger(t) =$

$-\widetilde{\Delta}(\mu^{\overline{\lambda}}(t)) \leq 0$ for $t \geq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$. So

$$F(V^\dagger, t) = r\frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} + \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}\left(1 - \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\right)\overline{V} - \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V}.$$

Now use Lemma O.4 to eliminate $\dot{\mu}^{\overline{\lambda}}(t)$ from the rhs. The result, after simplifying, is $F(V^\dagger, t) = 0$, as desired.

As $V^\dagger$ is a bounded absolutely continuous function, it follows by a standard verification argument that $V^i(t) = V^\dagger(t)$ for $t \geq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$. Further, $\check{V}(\mu^{\overline{\lambda}}(t)) - c - V^i(t) = -\widetilde{\Delta}(\mu^{\overline{\lambda}}(t)) < 0$ for $t > (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$, hence firm i's unique optimal prospecting strategy for $t \geq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$ is $\lambda^i(t) = 0$.

Now consider times $t < (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$. If $\overline{\mu} = \pi_0$ then this time interval is empty, so assume $\overline{\mu} < \pi_0$. We will show that $V^\ddagger(t) \equiv \check{V}(\mu^{\overline{\lambda}}(t)) - c$ is a strict supersolution to the HJB equation for $t < (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$. That is, $F(V^\ddagger, t) > 0$ for all such times. This is sufficient to establish the unique optimality of the prospecting policy $\lambda^i(t) = \overline{\lambda}$ a.e. on $[0, \overline{T}]$, by the following argument. Recall that $\widetilde{\Delta}(\overline{\mu}) = 0$ by definition, and therefore by the definition of $\widetilde{\Delta}$

$$V^\ddagger((\mu^{\overline{\lambda}})^{-1}(\overline{\mu})) = \check{V}(\overline{\mu}) - c = \frac{\overline{\mu} - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} = V^i((\mu^{\overline{\lambda}})^{-1}(\overline{\mu})).$$

Then if $V^\ddagger$ is a strict supersolution of the HJB equation for $t \leq \overline{T}$, it must be that $V^\ddagger(t) > V^i(t)$ for all $t < \overline{T}$. The HJB equation then implies that firm i's unique optimal prospecting policy is $\lambda^i(t) = \overline{\lambda}$ for $t \leq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$.

As a first step toward establishing the supersolution result, note that $t \leq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$ implies $\widetilde{\Delta}(\mu^{\overline{\lambda}}(t)) \geq 0$ and therefore

$$V^\ddagger(t) = \check{V}(\mu^{\overline{\lambda}}(t)) - c \geq \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V}.$$

As $\mu^{\overline{\lambda}} > \pi_-$, this inequality implies $V^\ddagger(t) > 0$ for every $t \leq (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$.

First consider times $t \leq \min\{(\mu^{\overline{\lambda}})^{-1}(\overline{\mu}), (\mu^{\overline{\lambda}})^{-1}(\mu^*)\}$. On this time range $\check{V}(\mu^{\overline{\lambda}}(t)) - c = K(\mu^{\overline{\lambda}}(t) - \pi_A)$, and so $F(V^\ddagger, t)$ evaluates to

$$F(V^\ddagger, t) = rV^\ddagger(t) + \frac{\dot{\mu}^{\overline{\lambda}}(t)}{\pi_+ - \mu^{\overline{\lambda}}(t)}(\overline{V} - K(\pi_+ - \pi_A)).$$

By Lemma O.3, $\overline{V} \leq K(\pi_+ - \pi_A)$, meaning $F(V^\ddagger, t) > 0$ given $\dot{\mu}^{\overline{\lambda}}(t) < 0$ and $V^\ddagger(t) > 0$.

If $\overline{\mu} \geq \mu^*$ then we're done, so suppose instead that $\overline{\mu} < \mu^*$. Recall that for $t \in$

$((\mu^{\overline{\lambda}})^{-1}(\mu^*), (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})]$,

$$\check{V}(\mu^{\overline{\lambda}}(t)) = \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}h(\pi_+)(\pi_{++}R - 1) = \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}(K(\pi_+ - \pi_A) + c).$$

This expression, combined with the identity derived in Lemma O.4, allows us to evaluate $F(V^{\ddagger}, t)$ as

$$F(V^{\ddagger}, t) = -\overline{\lambda}\frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}(\overline{V} - K(\pi_+ - \pi_A)) - rc.$$

Now, $\mu^{\overline{\lambda}}(t) \in [\overline{\mu}, \mu^*)$ for $t \in ((\mu^{\overline{\lambda}})^{-1}(\mu^*), (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})]$, and therefore

$$\widetilde{\Delta}(\mu^{\overline{\lambda}}(t)) = \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}(\overline{V} - K(\pi_+ - \pi_A) - c) + c \leq 0,$$

or equivalently

$$-\overline{\lambda}\frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}(\overline{V} - K(\pi_+ - \pi_A)) \geq (\overline{\lambda} + r)c - \frac{\mu^{\overline{\lambda}}(t) - \pi_-}{\pi_+ - \pi_-}\overline{\lambda}c > rc.$$

This bound establishes that $F(V^{\ddagger}, t) > 0$ on $[(\mu^{\overline{\lambda}})^{-1}(\mu^*), (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})]$, as desired.

O.5.4 Proof of Lemma B.4

Note that c appears nowhere in the definition of Δ, and so μ^* is independent of c. First suppose that $\mu^* = \pi_0$. Note that $\widetilde{\Delta}(\pi_0)$ may be written

$$\widetilde{\Delta}(\pi_0) = h(\pi_0)\left(\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} - \max\left\{\pi_+ R - 1, \frac{\overline{\lambda}}{\overline{\lambda} + r}h(\pi_+)(\pi_{++}R - 1)\right\}\right) + c.$$

When $c \downarrow 0$, $\overline{V} \to \max\left\{\pi_+ R - 1, \frac{\overline{\lambda}}{\overline{\lambda}+r}h(\pi_+)(\pi_{++}R - 1)\right\}$. Thus the first term approaches a strictly negative value in this limit, while the second term approaches zero. This means $\widetilde{\Delta}(\pi_0) < 0$ for small c, i.e. $\overline{\mu} < \pi_0 = \mu^*$.

Next suppose $\mu^* < \pi_0$. In this case $\Delta(\mu^*) = 0$ and hence $\check{V}(\mu^*) = \frac{\mu^* - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda}+r}h(\pi_+)(\pi_{++}R - 1)$. So $\widetilde{\Delta}(\mu^*)$ may be written

$$\widetilde{\Delta}(\mu^*) = \frac{\mu^* - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\left(\overline{V} - h(\pi_+)(\pi_{++}R - 1)\right) + c.$$

$\overline{V}$ is decreasing in c, but due to time discounting $\overline{V} < h(\pi_+)(\pi_{++}R - 1)$ even in the limit as $c \downarrow 0$. So the first term is negative and bounded away from 0 for all c, meaning that for sufficiently small c, it must be that $\widetilde{\Delta}(\mu^*) < 0$. Hence $\overline{\mu} < \mu^*$ in this case as well.

Finally, note that when $c = 0$, π_A satisfies $h(\pi_A)(\pi_{A+}R - 1) = 0$, i.e. $\pi_{A+} = 1/R$. Hence $\pi_{A+} > \pi_-$ given that $\pi_{+-} < 1/R$. Also, when $c = 0$,

$$\widetilde{\Delta}(\pi_A) = \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}\overline{V} - \check{V}(\pi_A) \leq \frac{\pi_A - \pi_-}{\pi_+ - \pi_-}\frac{\overline{\lambda}}{\overline{\lambda} + r}(\overline{V} - h(\pi_+)(\pi_{++}R - 1)).$$

Note that

$$h(\pi_+)(\pi_{++}R - 1) > \max\left\{\pi_+R - 1, \frac{\overline{\lambda}}{\overline{\lambda} + r}h(\pi_+)(\pi_{++}R - 1)\right\} = \overline{V},$$

hence $\widetilde{\Delta}(\pi_A) < 0$ when $c = 0$. Thus $\overline{\mu} < \pi_A$ when $c = 0$, and so by continuity also for c sufficiently small.

O.5.5 Proof of Lemma B.5

Subsequent to time $\min\{T_i^*, \overline{T}_i\}$, firm $-i$ is in autarky with beliefs

$$\mu^{-i}(t) = \mu^{\overline{\lambda}}(\min\{T_i^*, \overline{T}_i\}) > \pi^A.$$

Thus its unique best reply at all such times is to prospect at rate $\overline{\lambda}$ and invest immediately. By Lemma O.9, it follows that firm $-i$'s unique optimal investment strategy is the cutoff rule $T_{-i}^* = \infty$. It remains only to characterize $-i$'s optimal prospecting behavior prior to time $\min\{T_i^*, \overline{T}_i\}$.

We proceed by establishing that $V^\dagger(t) = K(\mu^{-i}(t) - \pi_A)$ is a strict supersolution to firm $-i$'s HJB equation on $[0, \min\{T_i^*, \overline{T}_i\}]$. Recall that the HJB equation for firm $-i$ in this regime is

$$rV^{-i}(t) = \overline{\lambda}\left(K(\mu^{-i}(t) - \pi_A) - V^{-i}(t)\right)_+ - \frac{\dot{\mu}^{-i}(t)}{\pi_+ - \mu^{-i}(t)}\left(\overline{V} - V^{-i}(t)\right) + \dot{V}^{-i}(t).$$

So define the functional

$$F(w, t) \equiv rw(t) - \overline{\lambda}\left(K(\mu^{-i}(t) - \pi_A) - w(t)\right)_+ + \frac{\dot{\mu}^{-i}(t)}{\pi_+ - \mu^{-i}(t)}\left(\overline{V} - w(t)\right) - \dot{w}(t).$$

The claim that $V^\dagger$ is a strict supersolution is equivalent to $F(V^\dagger, t) > 0$ for $t < \min\{T_i^*, \overline{T}_i\}$.

Evaluating the functional at $V^\dagger$ yields

$$F(V^\dagger, t) = rV^\dagger(t) + \frac{\dot{\mu}^{-i}(t)}{\pi_+ - \mu^{-i}(t)}(\overline{V} - K(\pi_+ - \pi_A)).$$

Note that $\overline{V} \leq K(\pi_+ - \pi_A)$ by Lemma O.3, so the second term on the rhs is non-negative. Meanwhile $\mu^{-i}(t) > \pi_A$ for $t \leq \min\{T_i^*, \overline{T}_i\}$, so $F(V^\dagger, t) > 0$ as claimed.

Now note that as firm -i is in autarky at time $\min\{T_i^*, \overline{T}_i\}$, its value function at this point is

$$V^{-i}(\min\{T_i^*, \overline{T}_i\}) = \frac{\overline{\lambda}}{\overline{\lambda} + r} K(\mu^{-i}(\min\{T_i^*, \overline{T}_i\}) - \pi_A) < V^\dagger(\min\{T_i^*, \overline{T}_i\}).$$

This boundary condition combined with the fact that $V^\dagger$ is a strict supersolution implies $V^\dagger(t) > V^{-i}(t)$ for all $t \in [0, \min\{T_i^*, \overline{T}_i\}]$. The HJB equation then implies that $\lambda^{-i}(t) = \overline{\lambda}$ is firm $-i$'s unique best reply for all times.

O.5.6 Proof of Lemma B.6

If $\overline{\mu} = \pi_0$ then the result is automatic. So assume $\overline{\mu} < \pi_0$, in which case $\overline{\mu}$ is pinned down by the condition $\widetilde{\Delta}(\overline{\mu}) = 0$. If $\overline{\mu} \geq \mu^*$, then $\widetilde{\Delta}(\overline{\mu}) = 0$ may be written

$$\frac{\overline{\mu} - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r} \overline{V} - K(\overline{\mu} - \pi_A) = 0.$$

Note that the lhs is a difference of two terms. As $\overline{\mu} > \pi_-$, the first term is strictly positive, meaning the second must be as well given that their difference is zero. Hence $\overline{\mu} > \pi_A$.

Suppose instead that $\mu^* > \overline{\mu}$. In this case $\widetilde{\Delta}(\mu^*) < 0$, which is equivalently

$$\frac{\mu^* - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r} \overline{V} - K(\mu^* - \pi_A) < 0.$$

The lhs is again a difference of two terms. As $\mu^* > \pi_-$, the first term is strictly positive. Therefore the second term is as well, implying $\mu^* > \pi_A$.

O.5.7 Proof of Lemma B.7

Suppose that signals are substitutes for a particular choice of r and c. Then

$$\widetilde{\Delta}(\pi_0) = h(\pi_0) \left(\frac{\overline{\lambda}}{\overline{\lambda} + r} (\pi_+ R - 1) - \max\left\{ \pi_+ R - 1, \frac{\overline{\lambda}}{\overline{\lambda} + r} h(\pi_+)(\pi_{++} R - 1) \right\} \right) + c.$$

Setting $c = \bar{c}$ and taking $r \to 0$ yields $\widetilde{\Delta}(\pi_0) \to l(\pi_0)\bar{c} > 0$. So it must be that for r sufficiently small, $\widetilde{\Delta}(\pi_0) > 0$ when $c = \bar{c}$, thus also when c is sufficiently close to $\bar{c}$ by continuity. Note also that for any finite r, signals are substitutes when c is sufficiently close to $\bar{c}$. Thus fixing r and then taking c close to $\bar{c}$ does not violate the assumption that signals are substitutes.

O.5.8 Proof of Lemma B.8

Recall that

$$\widetilde{\Delta}(\mu) = \frac{\mu - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r} \overline{V} - \check{V}(\mu) + c,$$

where $\check{V}(\mu)$ is not a function of c. When signals are substitutes, $\overline{V}$ doesn't depend on c either, so that $\partial\widetilde{\Delta}/\partial c = 1 > 0$. When signals are complements,

$$\overline{V} = \frac{\overline{\lambda}}{\overline{\lambda} + r} \left(h(\pi_+)(\pi_{++}R - 1) - c\right)$$

and so

$$\frac{\partial\widetilde{\Delta}}{\partial c} = -\frac{\mu - \pi_-}{\pi_+ - \pi_-} \left(\frac{\overline{\lambda}}{\overline{\lambda} + r}\right)^2 + 1 > 0.$$

so $\widetilde{\Delta}$ is strictly increasing in c for every μ in all cases. Thus $\overline{\mu}$ is increasing in c, and strictly increasing whenever $\overline{\mu} < \pi_0$.

O.6 Proofs of auxiliary lemmas related to Proposition 4

O.6.1 Proof of Lemma B.9

Recall that $T^A = (\mu^{\overline{\lambda}})^{-1}(\pi_A)$ while $\overline{T} = (\mu^{\overline{\lambda}})^{-1}(\overline{\mu})$, so to first order in r^{-1},

$$T^A - \overline{T} = -\frac{1}{\dot{\mu}^{\overline{\lambda}}(T^A)}(\overline{\mu} - \pi_A) + O(r^{-2}).$$

Next, for large r $\overline{\mu}$, solves

$$\frac{\mu - \pi_-}{\pi_+ - \pi_-} \frac{\overline{\lambda}}{\overline{\lambda} + r}(\pi_+ R - 1) = h(\mu)(\mu_+ R - 1) - c.$$

Recall the representation $h(\mu)(\mu_+ R - 1) - c = K(\mu - \pi_A)$ derived in Lemma O.2. This is therefore a linear equation in μ, with solution

$$\overline{\mu} = \frac{K\pi_A - \frac{J\pi_-}{\overline{\lambda}+r}}{K - \frac{J}{\overline{\lambda}+r}},$$

where $J \equiv \overline{\lambda}(\pi_+ R - 1)/(\pi_+ - \pi_-)$. To first order in r^{-1},

$$\overline{\mu} = \pi_A + \frac{\frac{J}{K}(\pi_A - \pi_-)}{\overline{\lambda} + r} + O(r^{-2}).$$

Thus

$$T^A - \overline{T} = -\frac{\overline{\lambda}(\pi_A - \pi_-)}{(\pi_+ - \pi_-)\dot{\mu}^{\overline{\lambda}}(T^A)} \frac{\pi_+ R - 1}{K(\overline{\lambda} + r)} + O(r^{-2}).$$

Now, Lemma O.4 implies the identity

$$\frac{\dot{\mu}^{\overline{\lambda}}(T^A)}{\pi_+ - \pi_A} = -\overline{\lambda}\frac{\pi_A - \pi_-}{\pi_+ - \pi_-}.$$

Hence

$$T^A - \overline{T} = \frac{\pi_+ R - 1}{K(\pi_+ - \pi_A)(\overline{\lambda} + r)} + O(r^{-2}) = \frac{\pi_+ R - 1}{(h(\pi_+)(\pi_{++} R - 1) - c)(\overline{\lambda} + r)} + O(r^{-2}).$$

So

$$\lim_{r\to\infty} r(T^A - \overline{T}) = \lim_{r\to\infty} \frac{\pi_+ R - 1}{h(\pi_+)(\pi_{++} R - 1) - c} \frac{r}{\overline{\lambda} + r} = \frac{\pi_+ R - 1}{h(\pi_+)(\pi_{++} R - 1) - c}.$$